EMA 310: Vectors and Mechanics

Unit 3b: The t-distribution

Learning Competencies

The learner will be able to:

1. Illustrate the t-distribution
2. Construct a t-distribution
3. Identify regions under the t-distribution corresponding to different t-values; and
4. Identify percentiles using the t-table.

Features of the t-distribution

- The t-distribution, just like the standard normal curve, is **bell-shaped** and **unimodal**. It is **symmetric** about *t=0*. However, its variance is greater than 1. This makes its wider and flatter in the middle.
- It has more area in its tails than that of the standard normal curve.
- Its shapes depends on the sample size *n*. As the sample size *n* becomes larger, the t-distribution gets closer to the standard normal distribution.

Comparison of Standard Normal Curve and the t-distribution

Comparison

Comparison of Standard Normal Curve and the t-distribution

- Statistical analysis on some studies which cannot be done using the normal distribution can be done using the t-distribution.
- The t-distribution is used with small samples taken from the population that is approximately normal.
- The z-statistic in the previous lessons uses the value of population variance while the t-statistic below uses the sample deviation especially when the population variance is unknown.
- The z-statistic is used when $n \geq 30$ while t-statistic is used when $n < 30$.

The t-distribution formula is:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

where

$\bar{x} = sample\ mean$

$\mu = population\ mean$

$s = standard\ deviation\ of\ the\ sample\ mean$

$n = sample\ sizes$

- To find a value in the Table of *t* Critical Values, there is a need to adjust the sample size *n* by converting it to degrees of freedom *df*.
- The number of degrees of freedom is equal to the number of the remaining values in a data set that are free to vary after one or more values have been deducted. In the case of the t-distribution, one value is deducted.

Hence the formula is:

$$df = n - 1$$

where

$df = degrees\ of\ freedom$

$n = sample\ size$

Worked examples

Example1

A student researcher wants to determine whether the mean score in mathematics of the 25 students in Grade 11 Hebrews is significantly different from the average of the school which is 89. The mean and the standard deviation of the scores of the students in Section Hebrews are 5 and 15, respectively. Assume a 95% confidence level.

Solution

Step1. Find the degrees of freedom.

$$df = n - 1$$
$$= 25 - 1$$
$$= 24$$

Step2. Find the critical value. Use the Table of t Critical Values. Confidence level is 95%.

$(1 - \alpha)100\% = 95\%$

$(1 - \alpha) = 0.95$

$\alpha = 0.05$ (area in two tails)

$\frac{\alpha}{2} = 0.025$ (area in one tail)

Look at 24 under the column headed *df*. Move to the right along the row until reaching the column headed 0.05, area in two tails or 0.025 for area in one tail.

The critical value is 2.064.

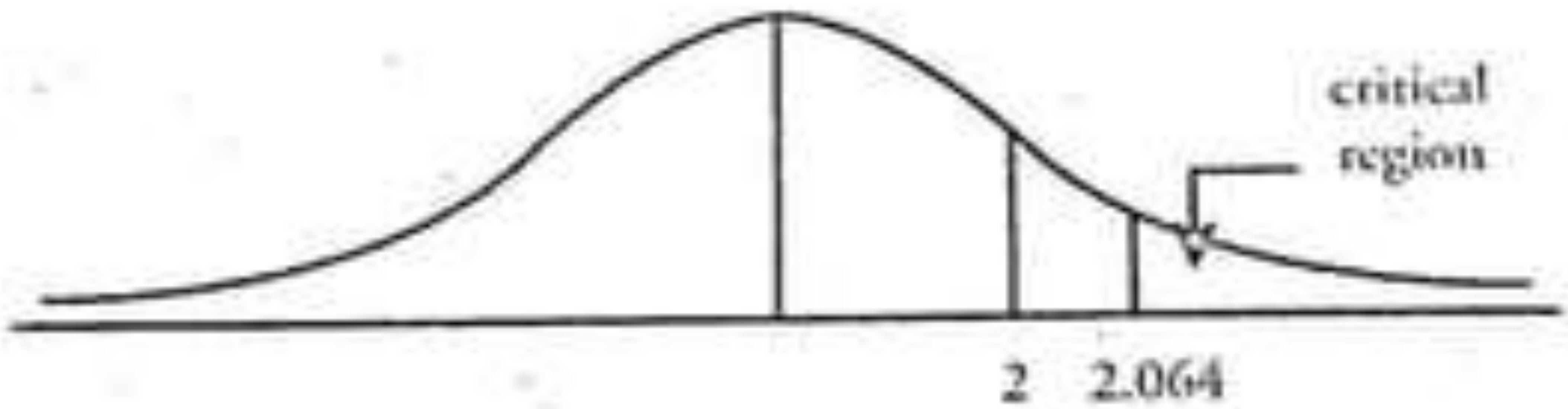
Step3. Compute the test statistic t.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$= \frac{95 - 98}{\frac{15}{\sqrt{25}}}$$

$$= 2$$

The computed value of t is equal to 2 which is smaller than the t-Critical value of 2.064



The value of the test statistic or computed *t* value does not fall in the critical region. Therefore, the mean score of Grade 11 section Hebrews in Mathematics is the same with mean score of all the students taking up Grade 11 Mathematics.

Example 2

A student suspects that the data she collected for research study do not represent the target population. Here are the data she collected.

16	27	34
20	29	30
22	30	37
25	30	42
26	32	35

The population mean is 27. Assuming normality in the target population, is the student's suspicion correct? Use a 90% confidence level.

Step1. Find the sample mean and the sample standard deviation.

Sample mean

$$\bar{x} = \frac{\sum x}{n} = \frac{435}{15} = 29$$

Sample variance

$$s^2 = \frac{(x - \bar{x})^2}{n - 1}$$

$$s^2 = \frac{634}{15 - 1}$$

$$s^2 = 45.29$$

Sample standard deviation

$$s = \sqrt{\frac{(x - \bar{x})^2}{n - 1}}$$

$$s = \sqrt{45.29} = 6.73$$

Observation	x	$x - \bar{x}$	$(x - \bar{x})^2$
1	16	16-29=-13	169
2	20	20-29=-9	81
3	22	22-29=-7	49
4	25	25-29=-4	16
5	26	26-29=-3	9
6	27	27-29=-2	4
7	29	29-29=0	0
8	30	30-29=1	1
9	30	30-29=1	1
10	32	32-29=3	9
11	34	34-29=5	25
12	30	30-29=1	1
13	37	37-29=8	64
14	42	42-29=13	169
15	35	35-29=6	36
	$\sum x = 435$		$\sum (x - \bar{x})^2 = 634$

Step2. Find the degrees of freedom.

$$df = n - 1$$
$$= 15 - 1$$
$$= 14$$

Step3. Find the critical value. Confidence level is 90%.

$$(1 - \alpha)100\% = 90\%$$
$$(1 - \alpha) = 0.90$$
$$\alpha = 0.10$$
$$\frac{\alpha}{2} = 0.05$$

Use of *t*-Critical Values Table.

In the column headed *df*, look at 14. Move to the right until the column headed 0.05 for one tail and 0.10 for two tails is reached. The intersection is 1.761. Hence, the critical value is 1.761.

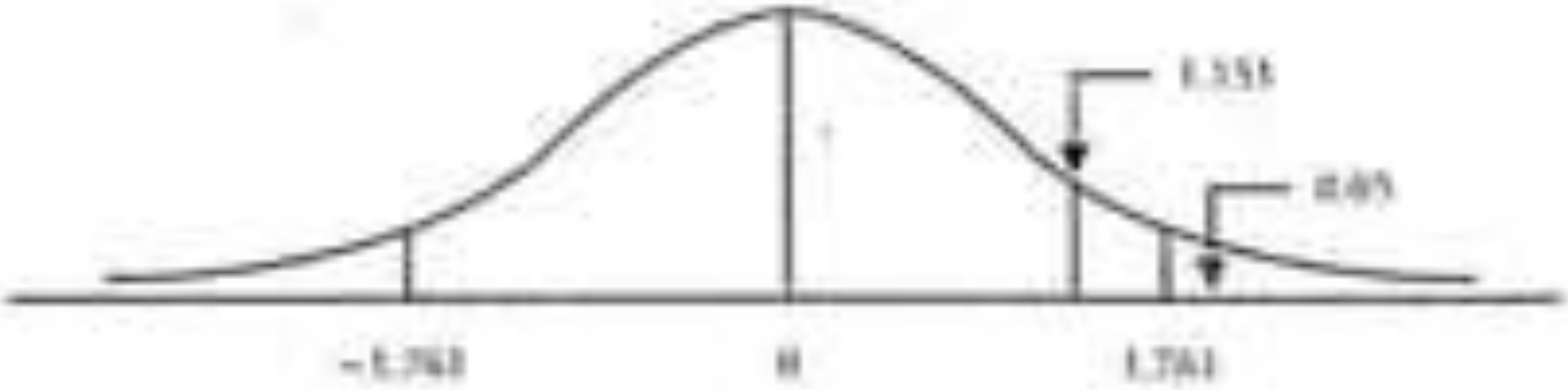
Step4. Compute the test statistic *t*.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$t = \frac{29 - 27}{\frac{6.73}{\sqrt{15}}}$$

$$t = \frac{2}{\frac{6.73}{\sqrt{15}}}$$

$$t = 1.151$$



The value of test statistic or computed t-value is less than the tabular value of 1.761. Therefore, the student is wrong in suspecting that the data are not representative of the target population.

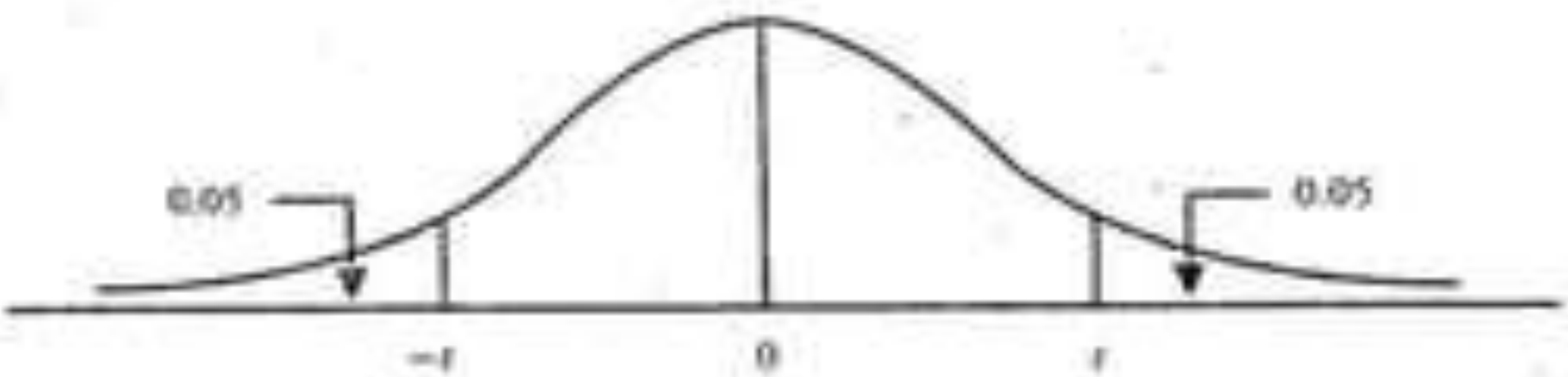
Example 3

A sample of size n=20 is a simple random sample selected from a normally distributed population. Find the value of t-such that the shaded area to the **left of t is 0.05**.

Solution

a. Find the degrees of freedom *df*.

$$df = n - 1$$
$$= 20 - 1$$
$$= 19$$



Since the t-distribution is symmetric about 0, then the area to the right of t is 0.05 and to the left of t is also 0.05. In the Table of t-Critical Values, move down the first column headed *df* until the *df* until *df=19*. Move to the right along this row reaching the column headed 0.05 (area in one tail) or 0.10 (area in two tails).

$$-t_{0.05} = -1.729$$

Since we are interested to the area at the left of *t*, hence, the negative value.

Example 4

Suppose you have a sample of size n=12 from a normal distribution. Find the critical value $t_{\frac{\alpha}{2}}$ that corresponds to a 95% confidence level.

Solution

a. Find the degrees of freedom *df*

$$\begin{aligned} df &= n - 1 \\ &= 12 - 1 \\ &= 11 \end{aligned}$$

b. Confidence level is 95%

$$(1-\alpha)100\% = 95\%$$

$$(1-\alpha) = 0.95$$

$$\alpha = 0.05$$

$$\frac{\alpha}{2} = 0.025$$

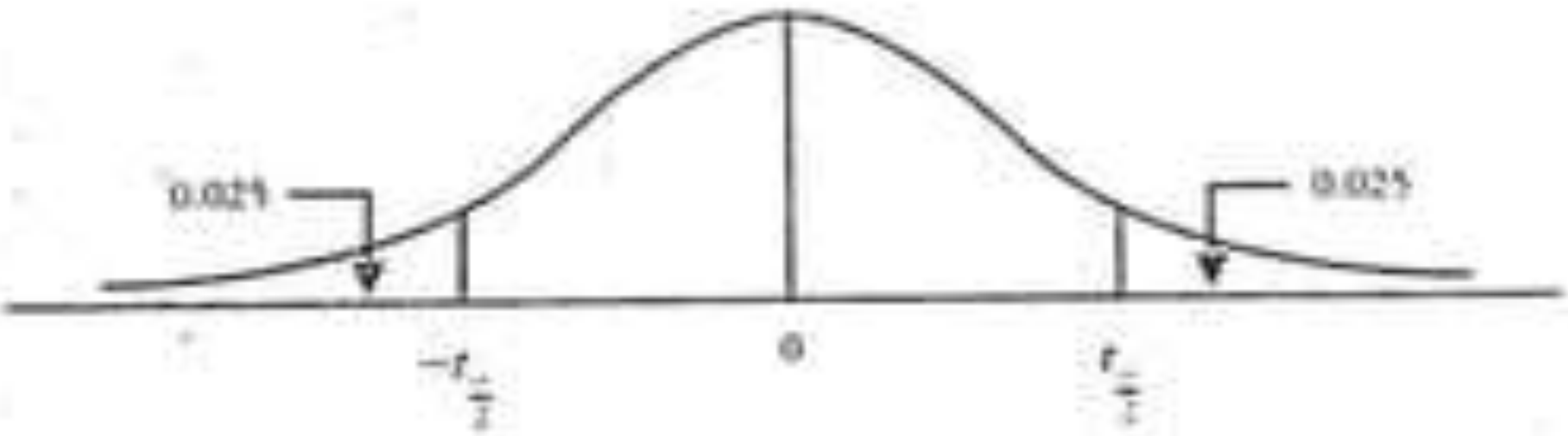
Solution

a. Find the degrees of freedom *df*

$$df = n - 1$$
$$= 12 - 1$$
$$= 11$$

b. Confidence level is 95%

$$(1 - \alpha)100\% = 95\%$$
$$(1 - \alpha) = 0.95$$
$$\alpha = 0.05$$
$$\frac{\alpha}{2} = 0.025$$



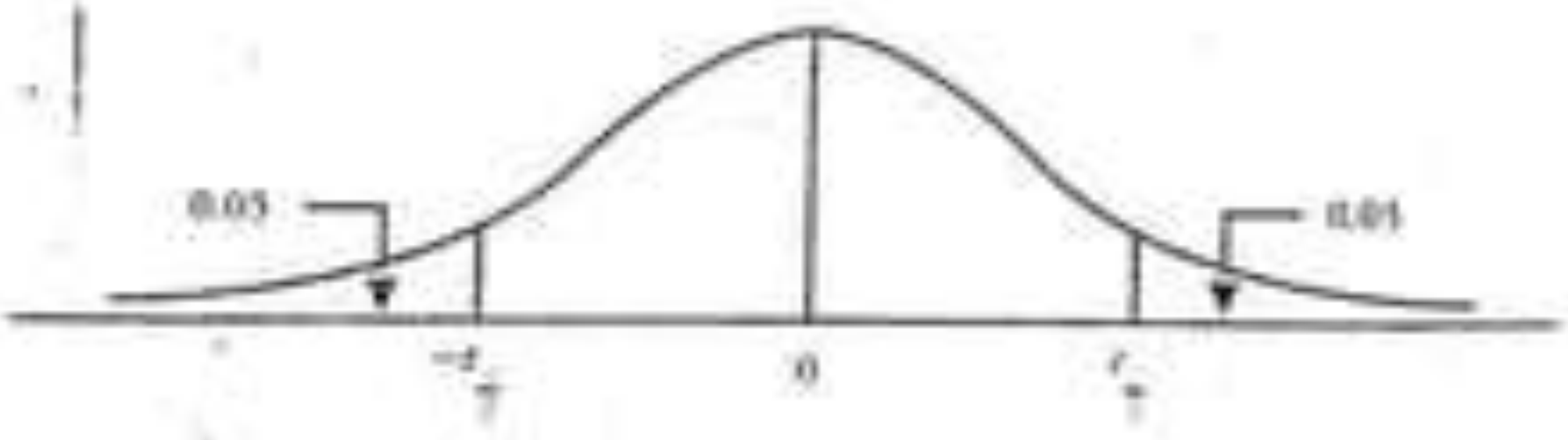
In the Table of *t*-Critical Values, move down the first column headed *df* until *df=11*, then move to the right along this row until the column headed 0.05 for two tails is reached.

Example 5

For a t-distribution with 25 degrees of freedom, find the values of t-such that the **area to the right of t is 0.05.**

Solution

a. The degrees of freedom *df*=25.



Using the Table of Critical Values of t, move down the column headed *df* until *df=25*. Move to the right along this row until reaching the column headed 0.05 for the area in two tails.

$$t_{\frac{\alpha}{2}} = t_{0.05} = 1.708$$

Example 6

For a t-distribution with 14 degrees of freedom, find the value of t such that the area between –t and t is 0.90.

Solution:

a. Find the degrees of freedom *df=14*.

b.

$(1-\alpha)100\% = 90\%$

$(1-\alpha) = 0.90$

$\alpha = 0.10$

$\frac{\alpha}{2} = 0.05$

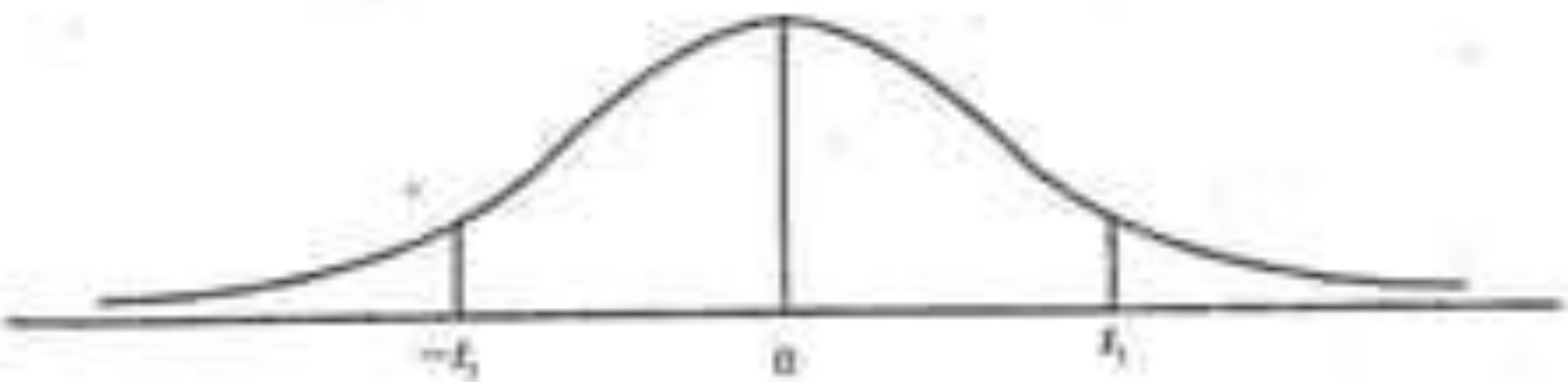
Using the Table of Critical Values of t, move down the column headed *df* until *df=14*. Move to the right along this row until reaching the column headed 0.10 for the area in two tails.

$$t_{\frac{\alpha}{2}} = t_{0.05} = 1.761$$

IDENTIFYING PERCENTILES USING THE t-DISTRIBUTION TABLE

Example 7

The graph of a distribution with *df*=25 is shown below.



a. If the shaded area on the right is 0.05, what is the area to the left of t_1?
b. What does t_1 represent?
c. Find the value of t_1.

Solution

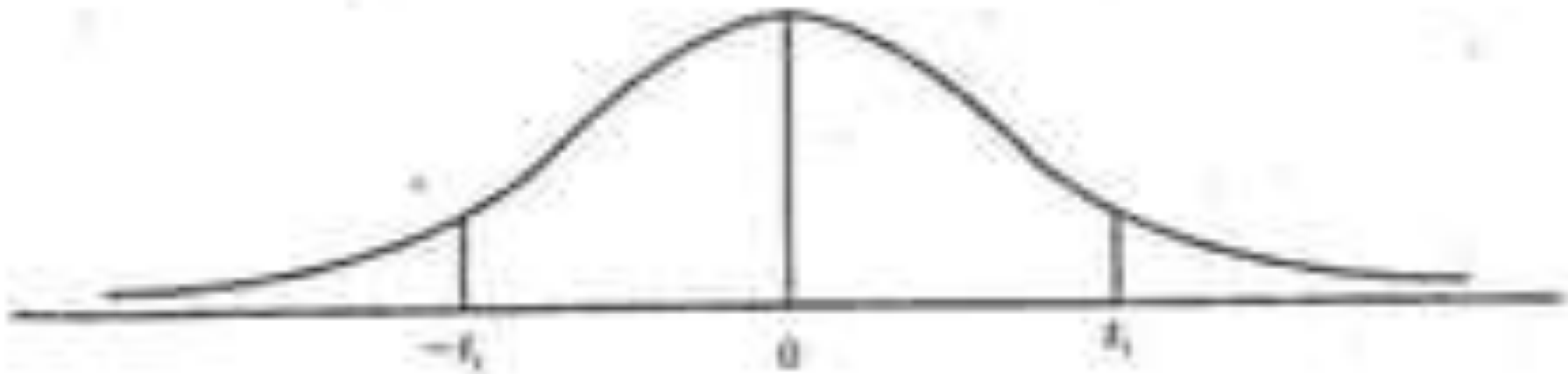
a. $(1-\alpha)100\% = \text{ρ}$
$(1-0.05) = \rho$
$0.95(100\%) = \rho$
$\rho = 95\%$
$\rho = 0.95\ area\ to\ the\ left\ of\ t_1$

b. Hence t_1 represents the 95th percentile $t_{0.95}$

c. To find the value of t_1, look at 18 under the column headed *df*. Proceed to the right until the column headed 0.05 for one tail is reached. The result is 1.753.

Example 8

The graph of a t-distribution with 18 *df* is shown below.



a. If the total shaded area of the curve is 0.02, what is the area to the left of t_1?

b. What does t_1 represent?

c. What is the value of t_1?

Solution

a. $(1-\alpha)100\% = \rho$

$$\left(1-\frac{0.02}{2}\right)100\% = \rho$$

$$1-0.01 = \rho$$

$$\rho = 0.99$$

b. t_1 represents the 99th percentile $t_{0.99}$

c. To find the value of t_1, look at 18 under column headed *df*. Proceed to the right until the column headed 0.01 for one tail is reached. The result is 2.552.