# Module for B.Ed Primary/Junior High School Programme

**2nd Semester**
**April, 2023**

**IoE/MoF/TUC/GHANA CARES TRAINING AND RETRAINING PROGRAMME FOR PRIVATE SCHOOL TEACHERS**

Ministry of Finance

Trade Union Congress

Institute of Education, UCC

# EBS424: Vectors and Mechanics

# Unit 1: Meaning and representation of a vector

**Dr. Isaac Benning**
**Dept. of Maths & ICT Education**
**Faculty of Science and Technology Education**
**University of Cape Coast**

# Learning objectives

By the end of this unit, you should be able to:

1. explain the meaning of the term *vector;*
2. distinguish between scalar and vector quantities;
3. represent a vector in the diagram, component, or magnitude-bearing form;
4. resolve vectors into components and vice versa;
5. Describe types of vectors.

# Activity (Whole class discussion)

Indicate whether the following quantities is a scalar or vector. Give reason.

(a) A temperature of $100^0$C is ………quantity.

(b) An acceleration of 9.8 $m/s^2$ vertically downward is a …….quantity.

(c) The weight of a 7 kg is a ……. quantity

(d) The sum of £500 is a …….quantity

(e) A north-easterly wind of 20 knot is a ……quantity

# Introduction

- **Physical quantities** are a characteristic or property of an object that can be measured or calculated from other measurements.
- **Units** are standards for expressing and comparing the measurement of physical quantities

- Examples of physical quantities are mass, amount of substance, length, time, temperature, electric current, light intensity, force, velocity, density, etc.

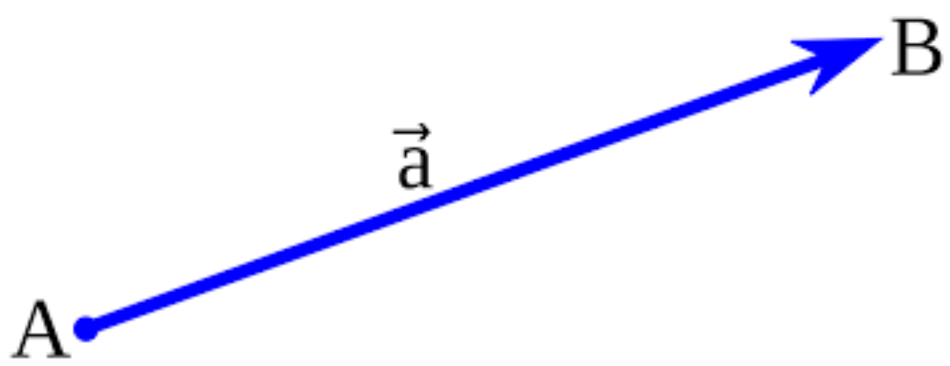
- Physical quantities can be divided into two main groups:
  - ***scalar quantity***
  - **vector quantity**

# Introduction

***(a)A scalar quantity*** is one that is defined completely by a single number with appropriate units, e. g., length, area, mass, volume, time, distance, etc. Once the unit is stated, the quantity is denoted entirely by its *size* or *magnitude*.

**(b)A vector quantity** is defined completely when we know not only its magnitude (with units) but also the direction in which it operates, e. g., force, velocity, displacement, acceleration, etc. A vector is usually described as a **quantity** that has a **magnitude** (**size**) and a **direction.**

# Representation of a Vector



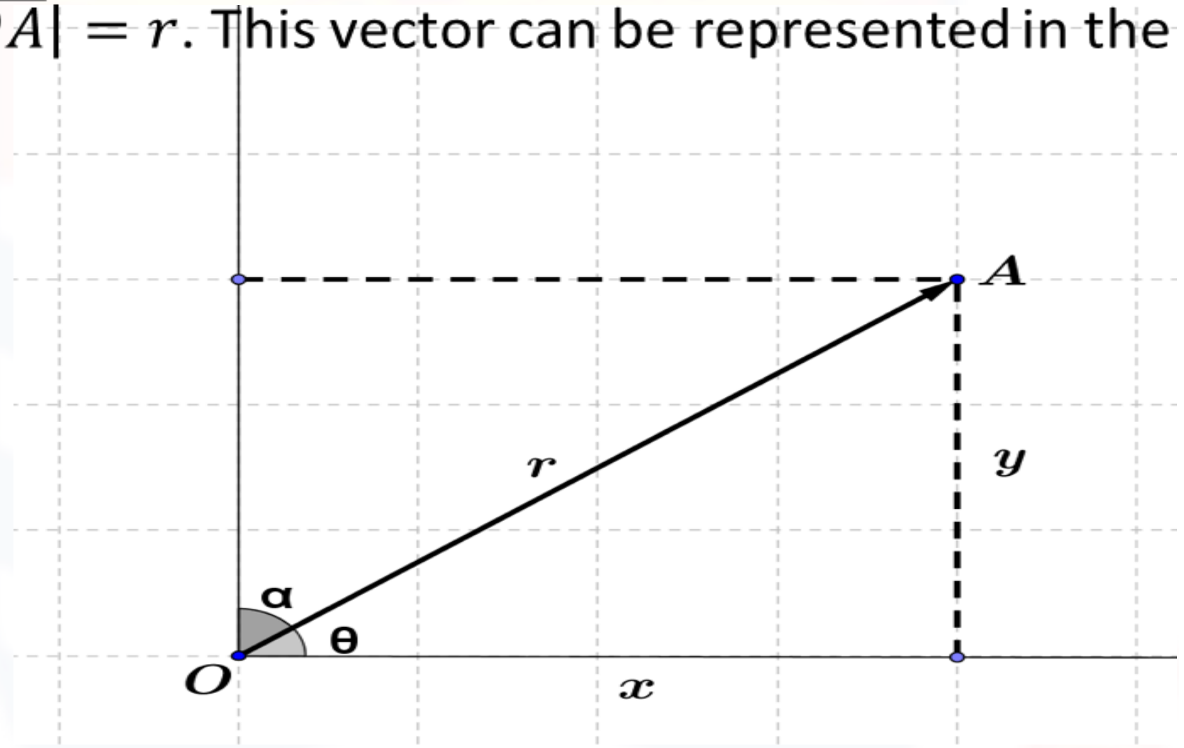


- Vectors are represented by lines with arrow.
- The **length of the line** indicates the **magnitude** of the vector, and the **direction** of the line indicates the **vector's direction**.
- An arrow is used to denote the sense of the vector, i.e. for a horizontal vector, say, whether it acts from left to right or vice versa.
- The arrow is positioned at the end of the vector and its position is called the **nose** of the vector

# Representation of a vector

A displacement from point $O$ to another point $A$ is a vector, written as $\overrightarrow{OA}$. The magnitude of vector $OA$ is $|\overline{OA}| = r$. This vector can be represented in the following ways:

(i) Diagram/graphical form




(ii) Component form $\overrightarrow{OA} = \begin{pmatrix} x \\ y \end{pmatrix}$ or $\overrightarrow{OA} = \begin{pmatrix} rcos\theta \\ rsin\theta \end{pmatrix}$

(iii) Cartesian form $\overrightarrow{OA} = xi + yj$

(iii) Magnitude-bearing form $\overrightarrow{OA} = (r, \alpha)$

The magnitude is $r = \sqrt{x^2 + y^2}$ and the bearing is α in the direction of $\overrightarrow{OA}$

# Resolution of vectors

Write the following vectors in the magnitude-bearing form.

$(a)\quad a = 4i + 3j$

$(b)\quad a = 5i - 4j$

$(c)\quad a = -2i + 3j$

$(d)\quad a = -6i - 4j$

# Resolution of vectors

Write the following vectors in the component form.

$(a)\quad a = (6N, 060^0)$

$(b)\quad a = (25\text{ km}, 120^0)$

$(c)\quad a = (7N, 180^0)$

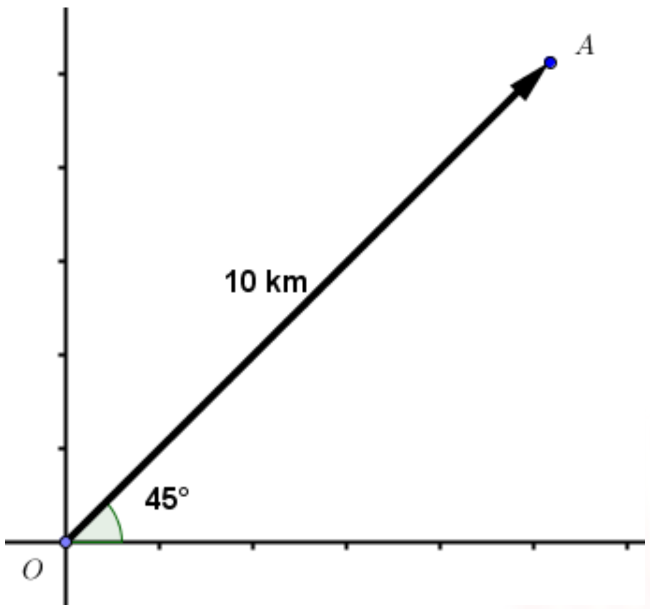
$(d)\quad a = (35N, 300^0)$

# Resolution of vectors

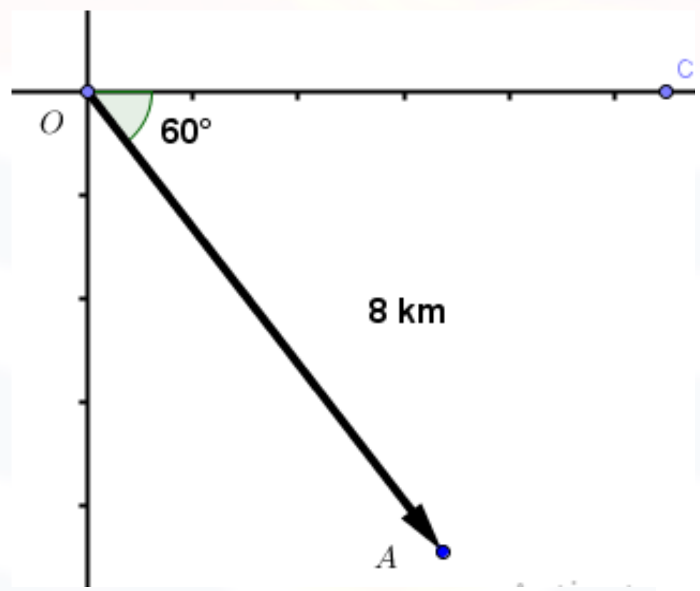
For each of the following diagrams, write the vector in the

(a) magnitude-bearing vector

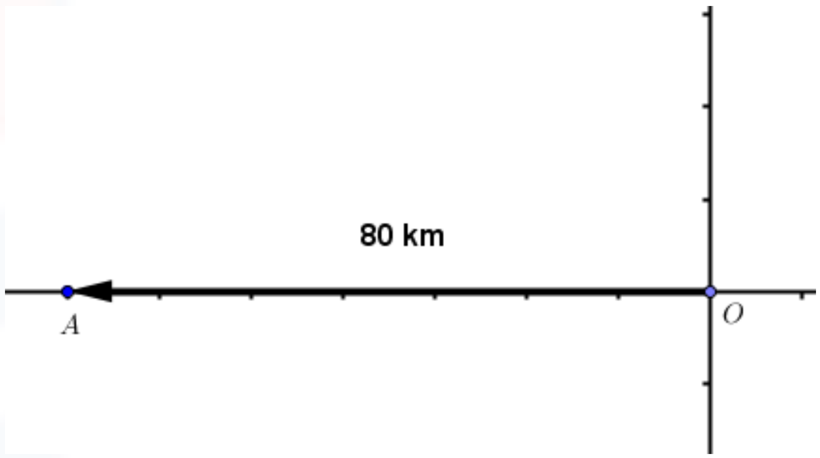
(b) Component form



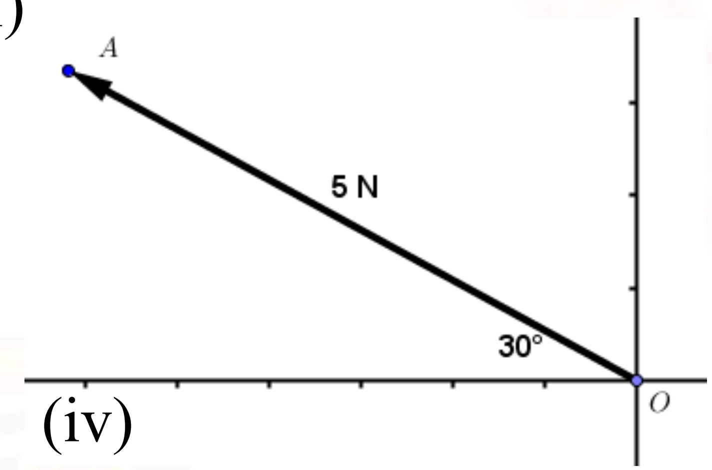

(i)



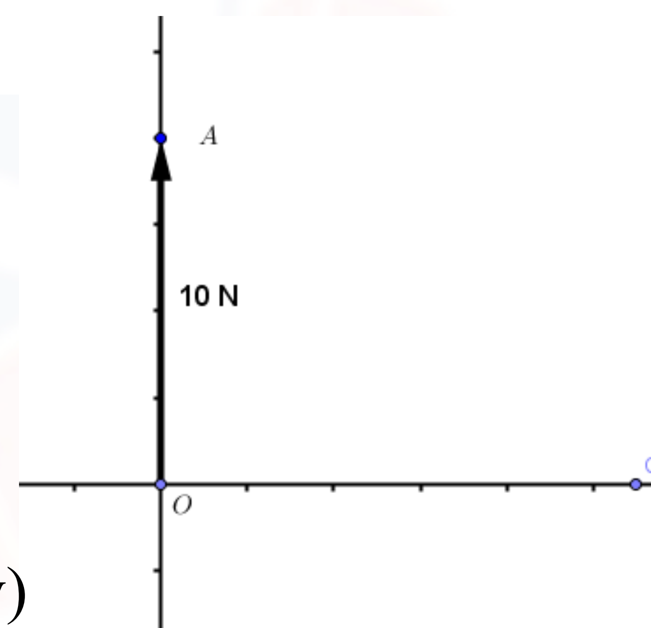

(ii)




(iii)

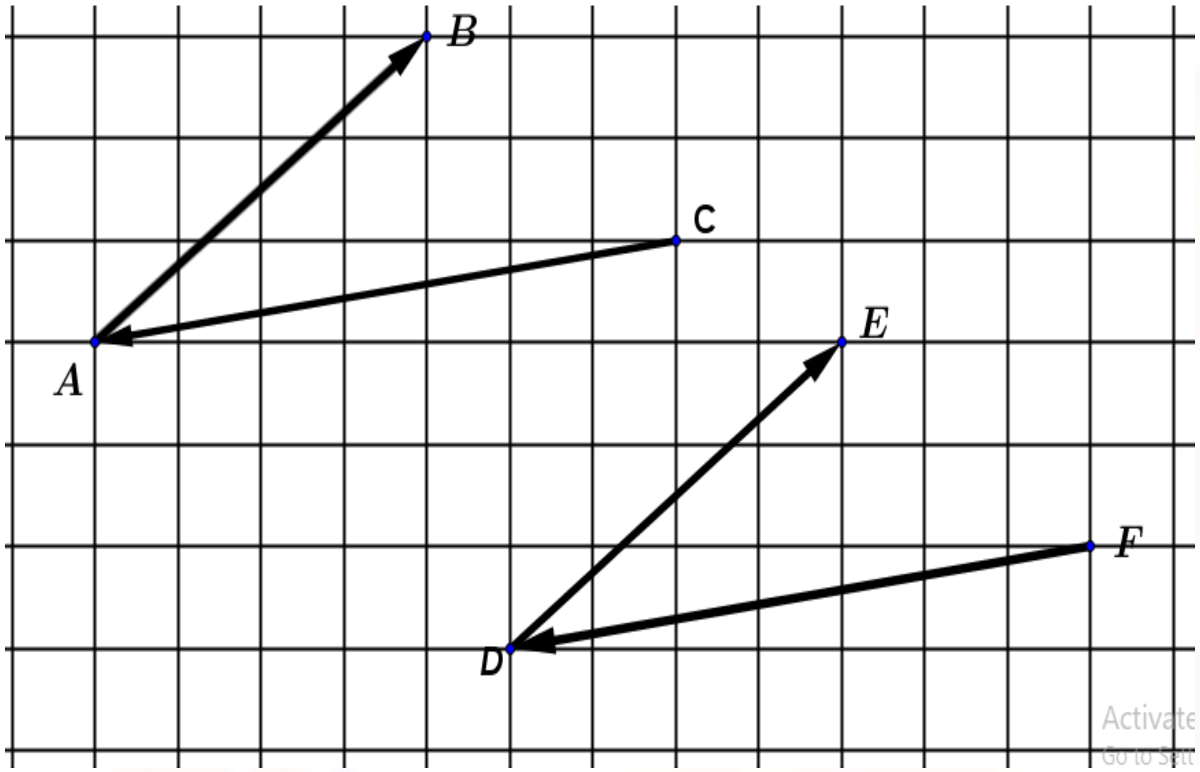



(iv)




(v)

# Free and Localised vectors




(1) Write the component form of the vectors:

  (i) $\overrightarrow{AB}$

  (ii) $\overrightarrow{CA}$

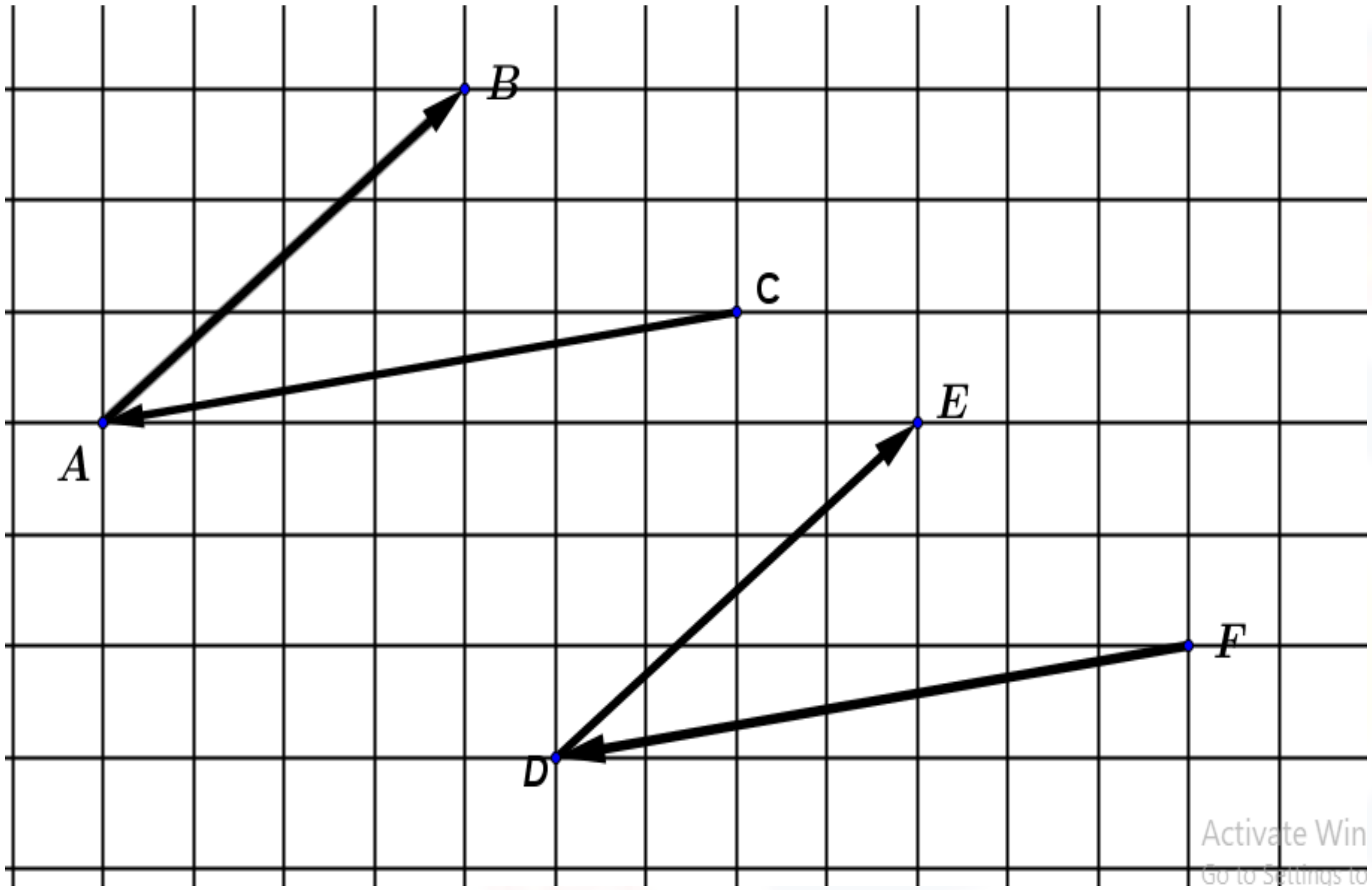
  (iii) $\overrightarrow{DE}$

  (iv) $\overrightarrow{FD}$

(2) Comment on vectors $\overrightarrow{AB}$ and $\overrightarrow{DE}$.

(3) Comment on vectors $\overrightarrow{CA}$ and $\overrightarrow{FD}$

# Free and Localised vectors




Write the component form of the vectors:

$(i)\ \overrightarrow{AB}$

$(ii)\ \overrightarrow{CA}$

$(iii)\overrightarrow{DE}$

$(iv)\overrightarrow{FD}$

Comments on vectors $\overrightarrow{AB}$ and $\overrightarrow{DE}$.

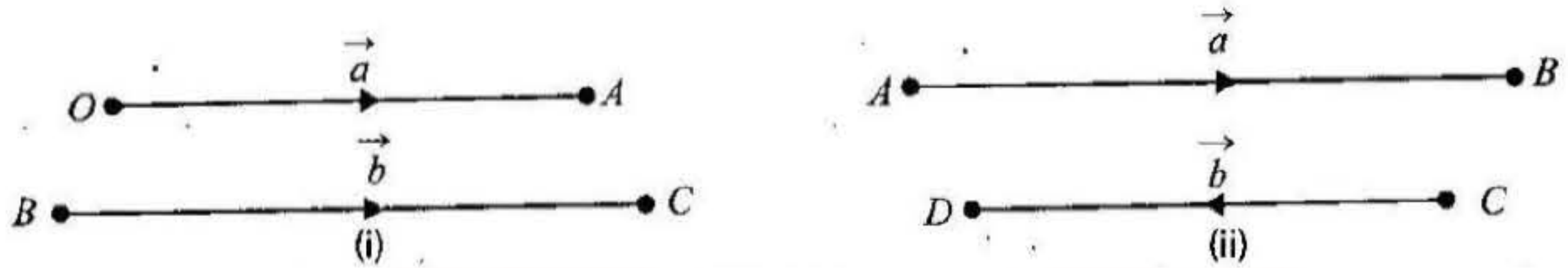
Comment on vectors $\overrightarrow{CA}$ and $\overrightarrow{FD}$

If we draw a vector or locate a vector in a particular position, we say we have a ***localized*** it, otherwise the vector is called a ***free vector***.

A **vector** which is drawn parallel to a **given vector** through a specified point in space is called a localized **vector.**

# Like and Unlike Vectors

- Two parallel vectors having the same direction are called like vectors (Fig i).
- Two parallel vectors having opposite directions are called unlike vectors (fig ii).

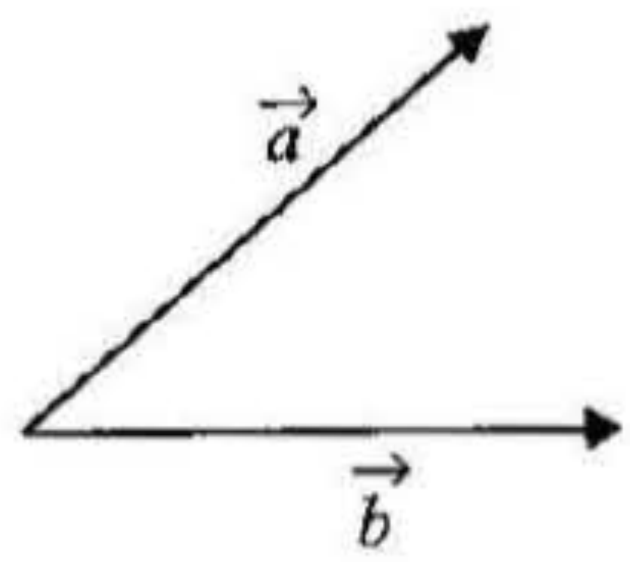


# Collinear Vectors

Vectors $\vec{a}$ and $\vec{b}$ are collinear if they have same direction or are parallel or anti-parallel. Since their magnitudes are different, we can find some scalar $\lambda$ for which $\vec{a} = \lambda \vec{b}$. If $\lambda > 0$, $\vec{a}$ and $\vec{b}$ are in the same direction; if $\lambda < 0$, $\vec{a}$ and $\vec{b}$ are in the opposite directions. Collinear vectors are often called dependent vectors.

# Non-collinear vectors

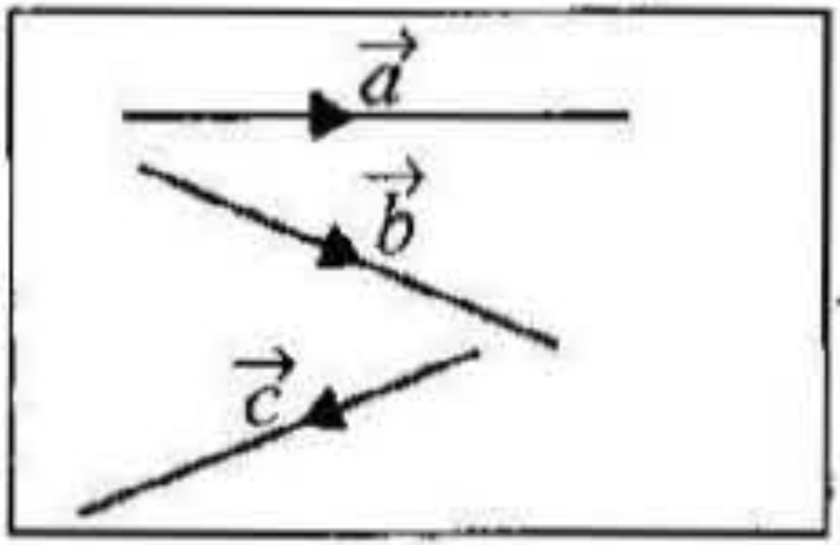
Two vectors acting in different directions are called non-collinear vectors. Non-collinear vectors are often called independent vectors. Here we cannot write vector $\vec{a}$ in terms of $\vec{b}$, though they have the same magnitude. However, we can find component of one vector in the direction of the other. Two non-collinear vectors describe plane.



# Coplanar Vectors

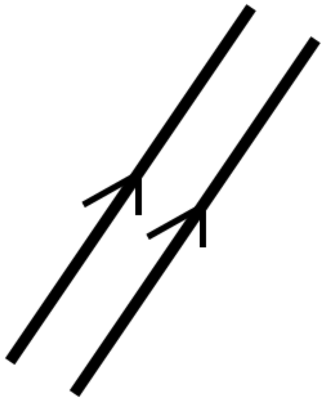
Two parallel vectors or non-collinear vectors are always coplanar or two vectors $\vec{a}$ and $\vec{b}$ in different directions determine unique plane in space. Now if vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors. Generally more than two vectors are coplanar if all are in the same plane.

Three non-coplanar vectors describe space.

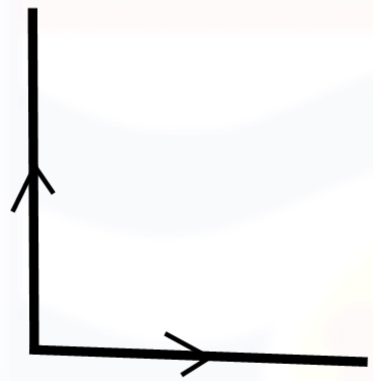


# Activity (Whole class discussion)

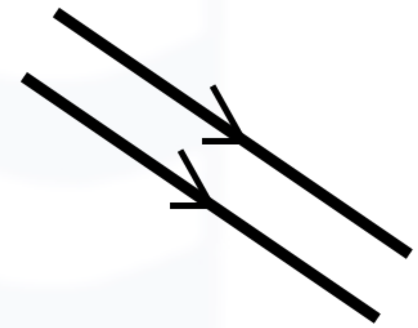
Which of the following pairs of vectors are equal?



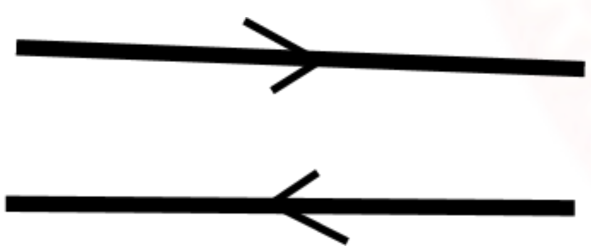
(i)



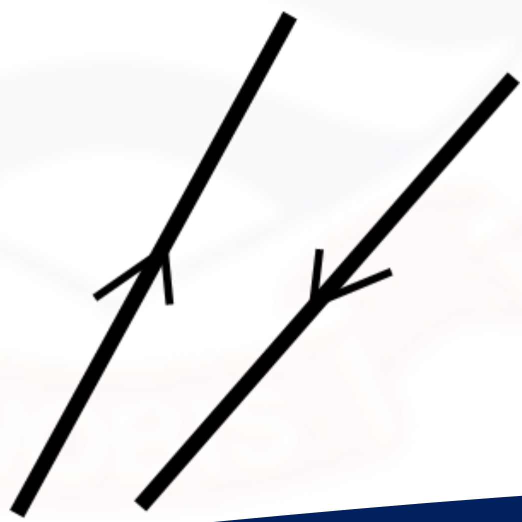
(ii)



(iii)



(iv)



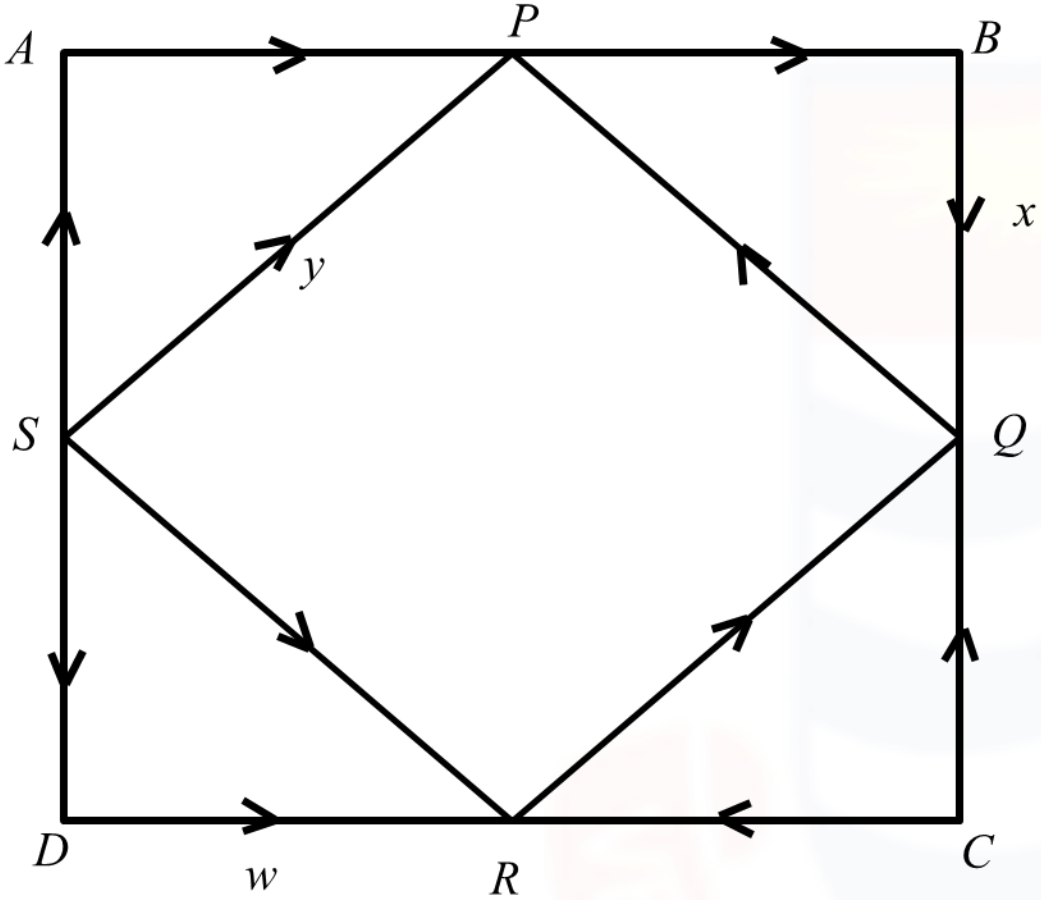
(v)

# Equal vectors

- Two vectors are (for example $\overrightarrow{AB}$ and $\overrightarrow{PQ}$)said to be equal (or equivalent) if they have the same magnitude and the same direction.

- Thus $\overrightarrow{AB} = \overrightarrow{PQ} \Leftrightarrow |AB| = |PQ|$ and $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ have the same direction.

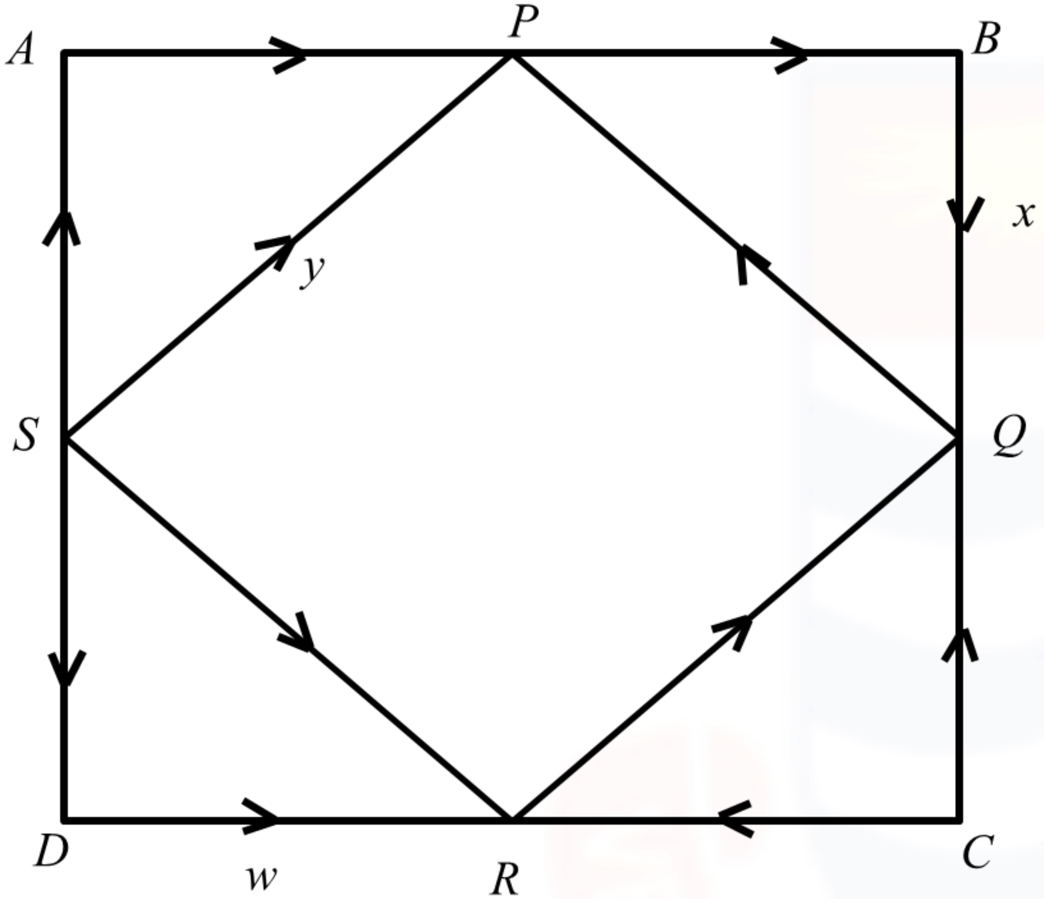
# Activity (Pair Work)




*ABCD* is a square. *P*, *Q*, *R*, *S* are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

1. Name the displacement vectors shown on the inner square.
2. Name the displacement vectors in the diagram which are equal $\overline{AP}$.
3. Which displacement vector may be written as ***w?***
4. Which displacement vectors are equal to ***x***?
5. Which displacement vectors have length $|y|$?
6. Which displacement vectors are equal to $\overline{RQ}$?

# Activity (Pair Work)-*Suggested Solution*




$ABCD$ is a square. $P, Q, R, S$ are the midpoints of $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$ respectively.

1. Name the displacement vectors shown on the inner square. $\overrightarrow{SP}, \overrightarrow{SR}, \overrightarrow{RQ}$, and $\overrightarrow{QP}$
2. Name the displacement vectors in the diagram which are equal $\overline{AP}$. $\overrightarrow{DR}$ and $\overrightarrow{PB}$
3. Which displacement vector may be written as ***w***? $\overrightarrow{DR}$
4. Which displacement vectors are equal to ***x***? $\overrightarrow{SD}$ $\overrightarrow{AS}$ and $\overrightarrow{QC}$
5. Which displacement vectors have length $|y|$? $\overrightarrow{SP}, \overrightarrow{SR}, \overrightarrow{RQ}$, and $\overrightarrow{QP}$
6. Which displacement vectors are equal to $\overline{RQ}$? $\overrightarrow{SP}$

# Negative Vector

- If $\boldsymbol{u}$ is a non-zero vector, the vector with the same magnitude but in the opposite direction to $\boldsymbol{u}$ is a called the negative of $\boldsymbol{u}$ and is denoted by $-\boldsymbol{u}$.

- That is $-\boldsymbol{u} = (-1)\boldsymbol{u}$.

- The negative the vector $\overrightarrow{AB}$ is $-\overrightarrow{AB}$.

- $-\overrightarrow{AB} = (-1)\overrightarrow{AB} = \overrightarrow{BA}$

# Summary

In this unit we looked at the differences between scalar and vector quantities and how vectors are represented in the diagram, component, or magnitude-bearing form. We also learned how to resolve vectors into components and vice versa and finally we described types of vectors. In unit 2, we will learn algebra of vectors.

# EBS424: Vectors and Mechanics

# Unit 2: Algebra of vectors

**Dr. Isaac Benning**
**Dept. of Maths & ICT Education**
**Faculty of Science and Technology Education**
**University of Cape Coast**

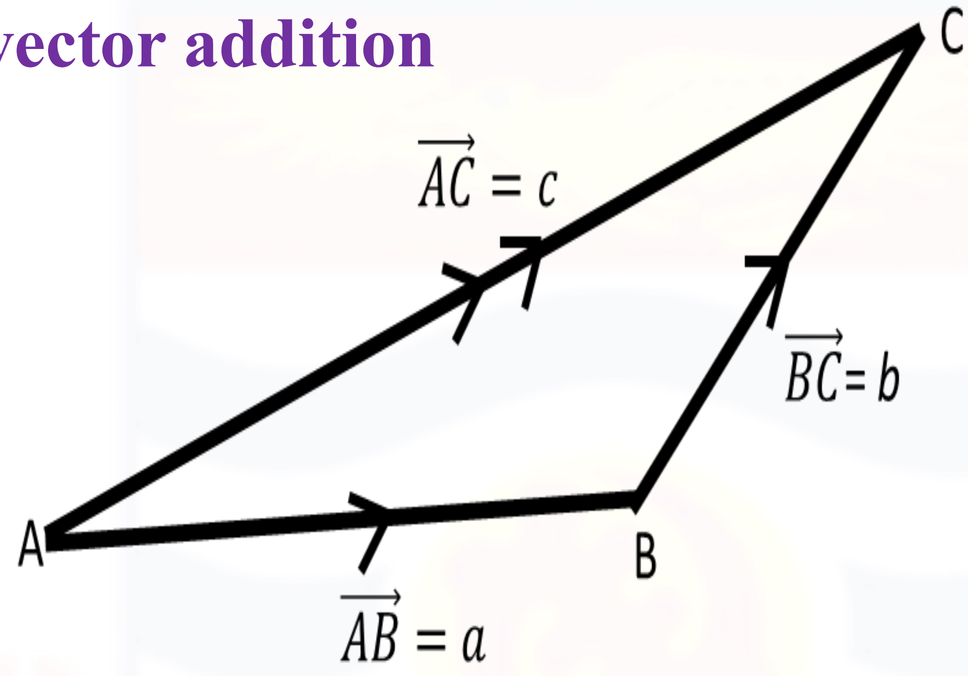
# Learning objectives

By the end of this unit, you should be able to:

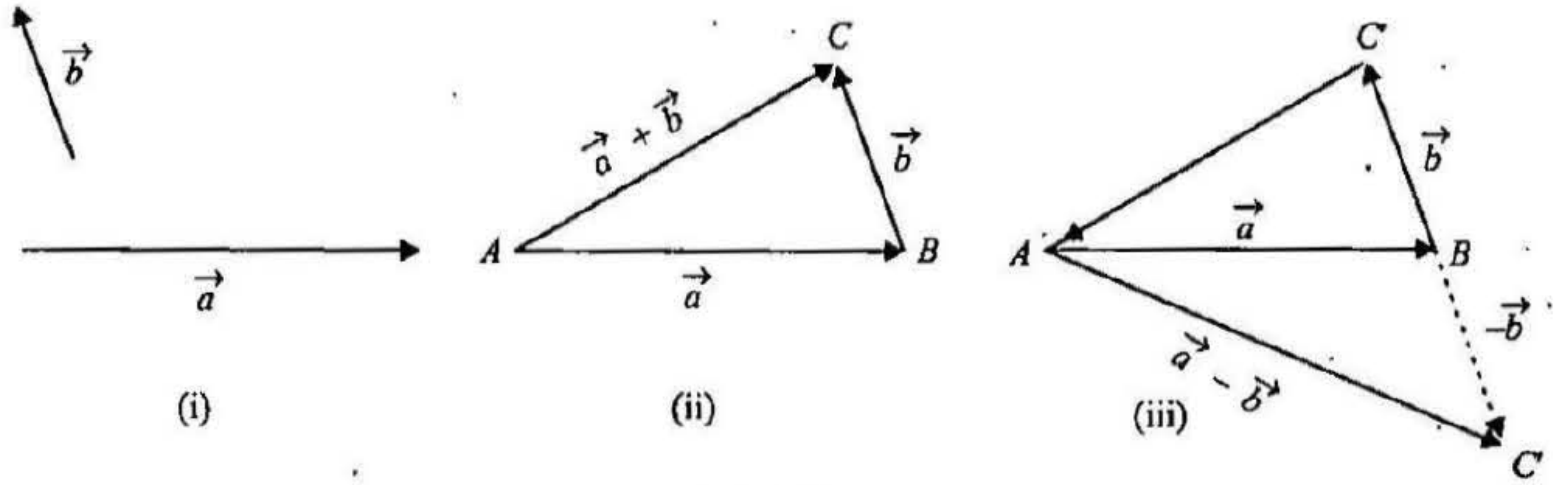
1. find the resultant of given vectors.
2. add vectors using parallelogram and triangle laws of addition.
3. establish and use properties of addition of vectors.

# Addition of Vectors

## a. The triangle law of vector addition

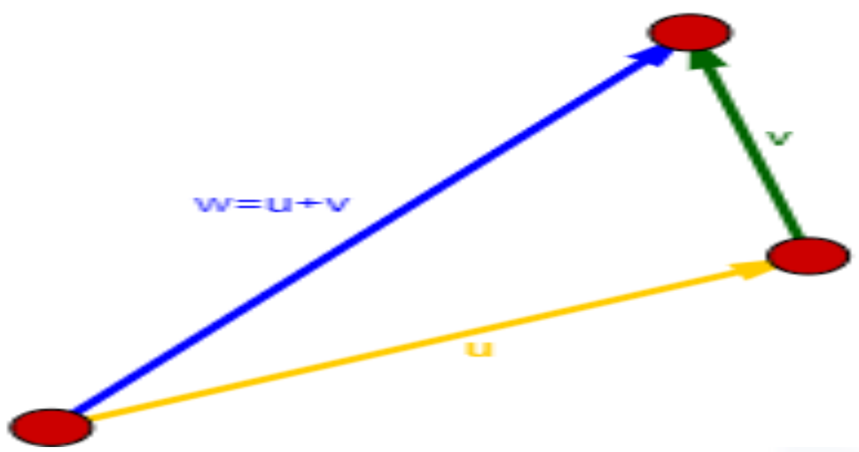


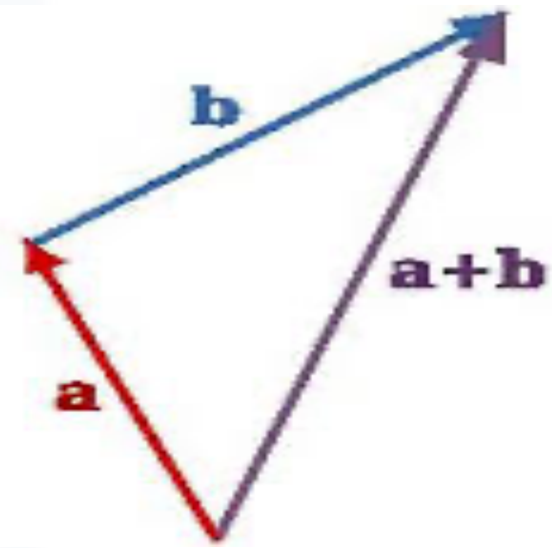
# Addition of Vectors

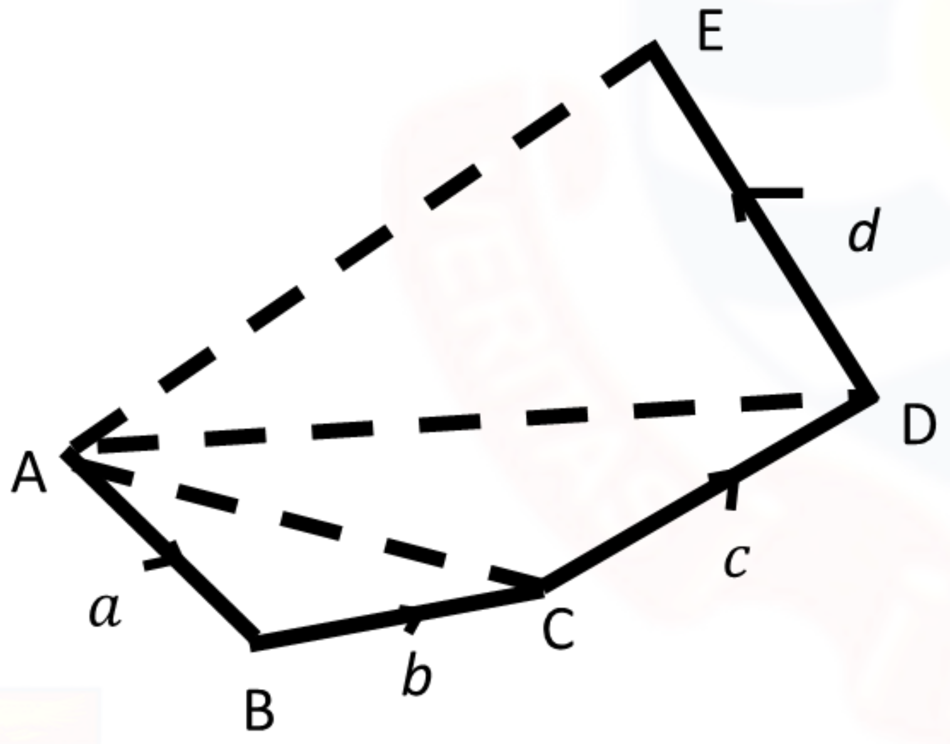


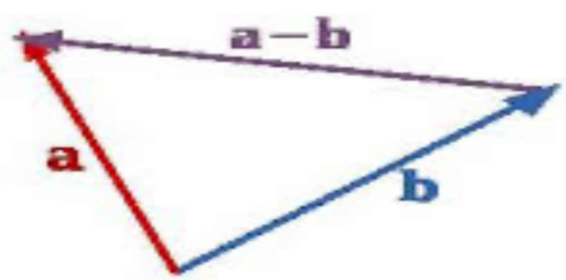
# Addition of Vectors

## a. The triangle law of vector addition



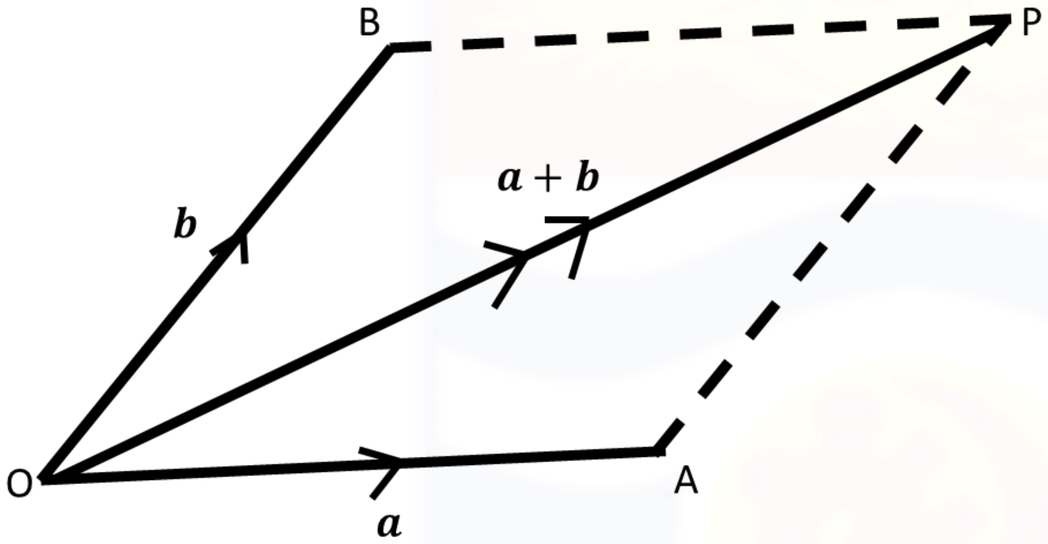



# Addition of Vectors

**b. The parallelogram law of vector addition**




Click to see animation

# Properties of Addition Vectors

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (commutative property)

2. $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (associative property)

3. $\vec{a} + \vec{0} = \vec{a}$ (additive identity)

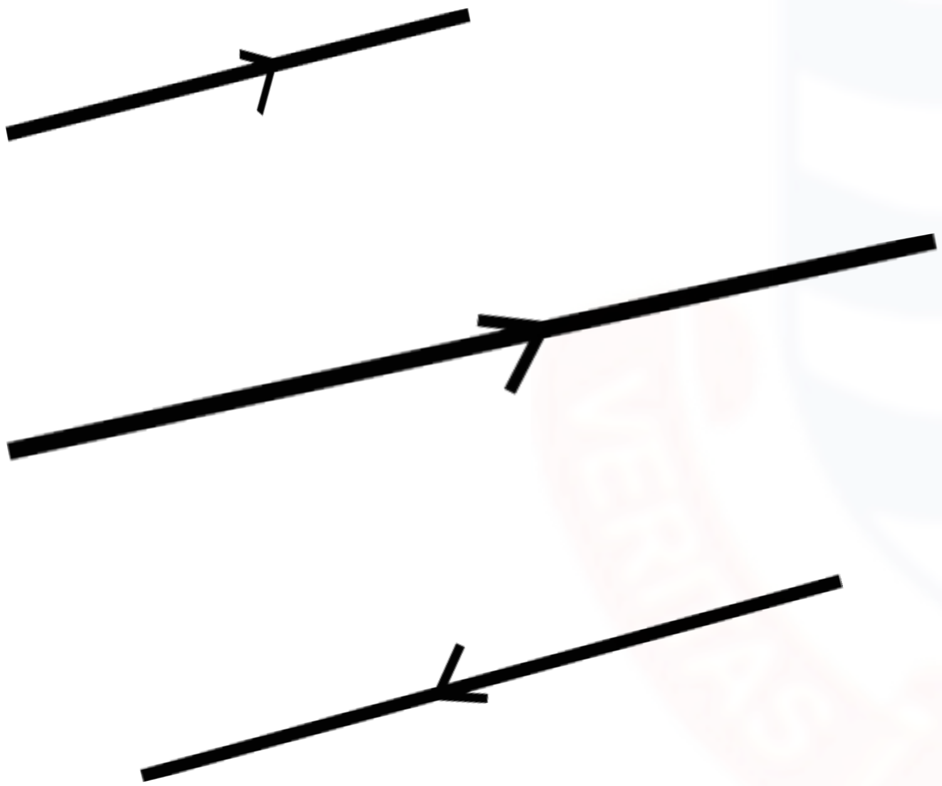
4. $\vec{a} + (-\vec{a}) = \vec{0}$ (additive inverse)

5. $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$ and $|\vec{a} - \vec{b}| \geq |\vec{a}| - |\vec{b}|$

# Activity (Individual)

Find the vector sum.

# Multiplication of a vector by a scalar



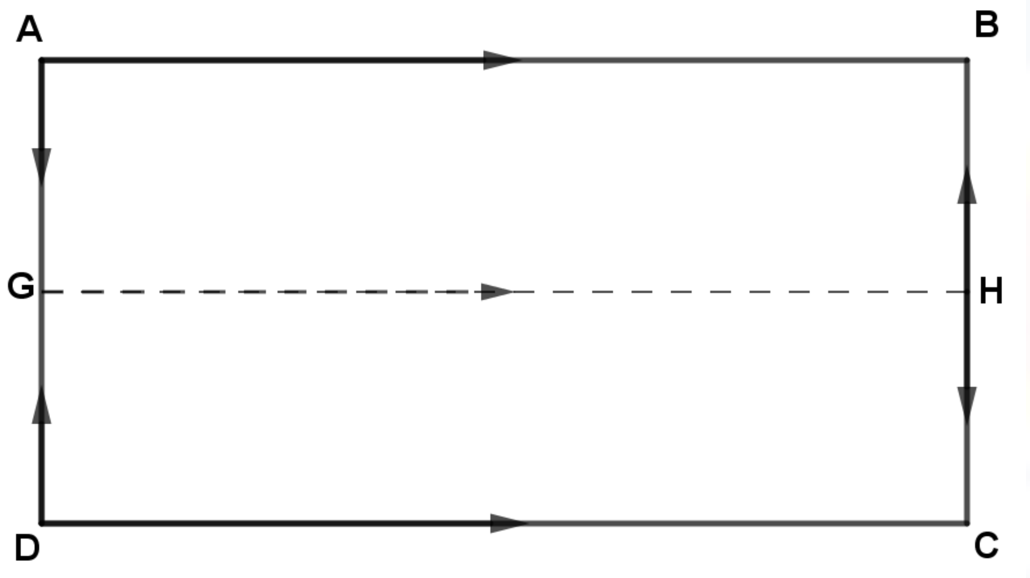
# Activity (Whole class discussion)

# Activity (Whole class discussion)

# Activity (Whole class discussion)

# Activity (Whole class discussion)

# Activity (Whole class discussion)



# Exercise

# EBS424: Vectors and Mechanics

## Unit 3: Position vector, Mid-point theorem, Unit vector and 3D vectors


**Dr. Isaac Benning**

**Dept. of Maths & ICT Education**

**Faculty of Science and Technology Education**

**University of Cape Coast**

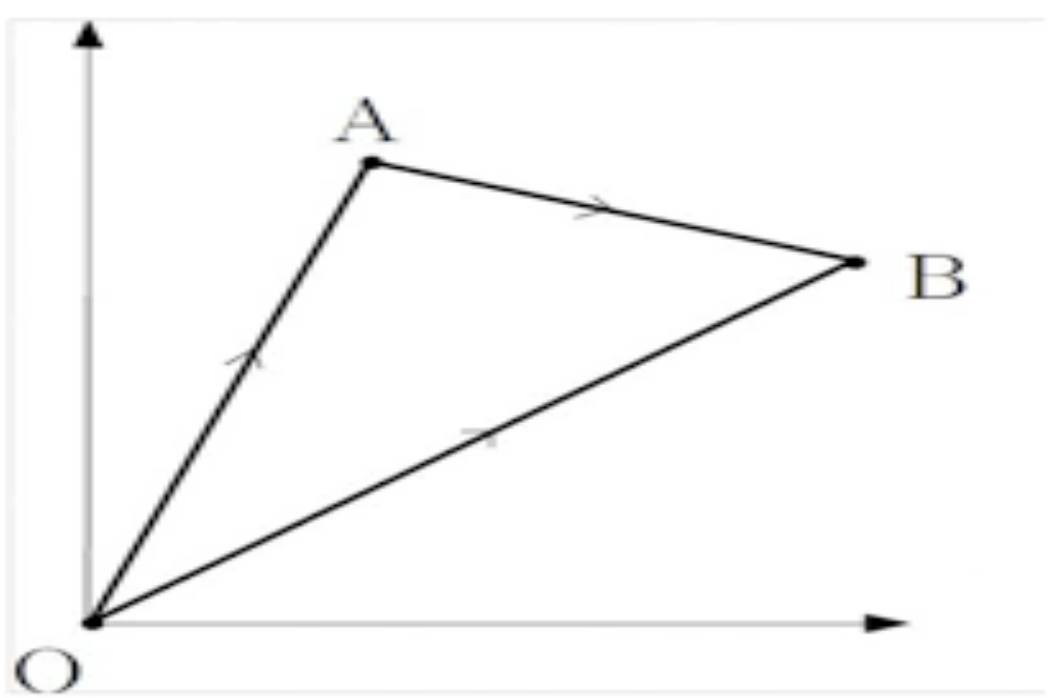
# Learning objectives

By the end of this unit, you should be able to:

1. express a given vector in terms of position vectors.
2. state and apply mid-point theorem
3. calculate the unit vector of a given vector.
4. describe vectors in 3D.

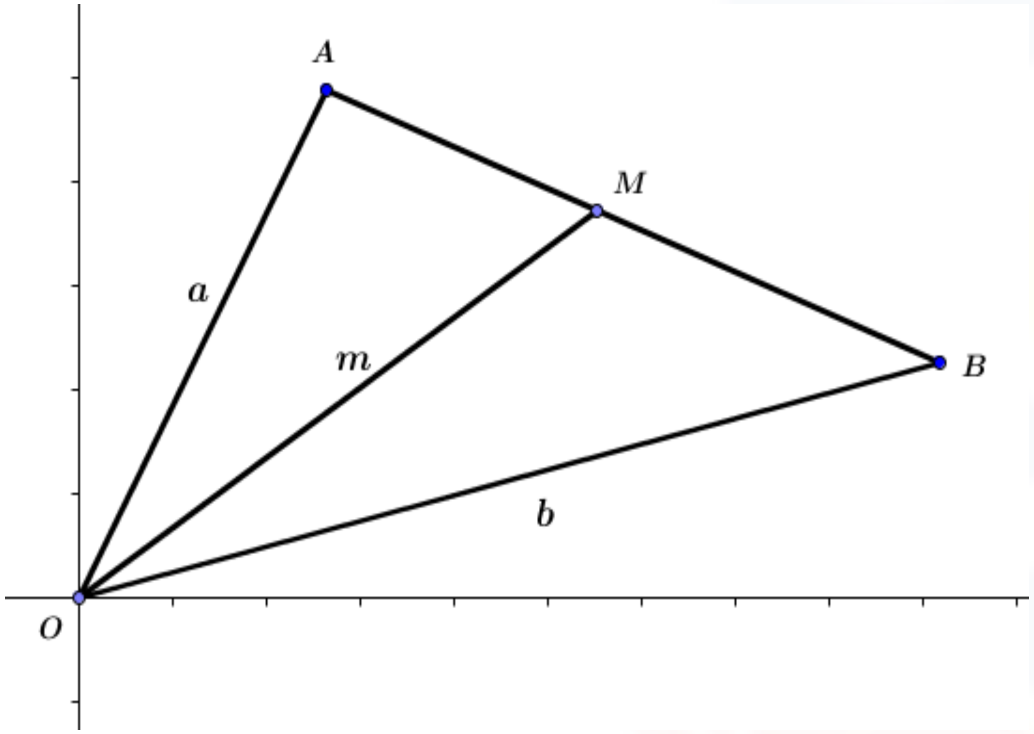
# Position Vector

**Proof**

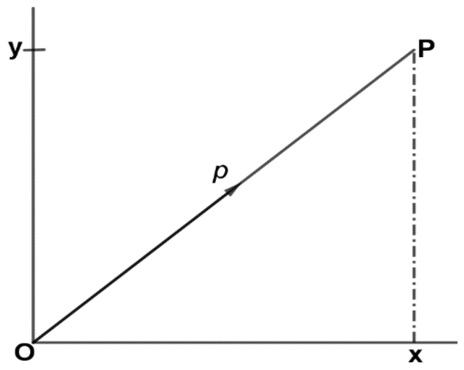


# Midpoint Theorem

**Proof**



# Unit Vector

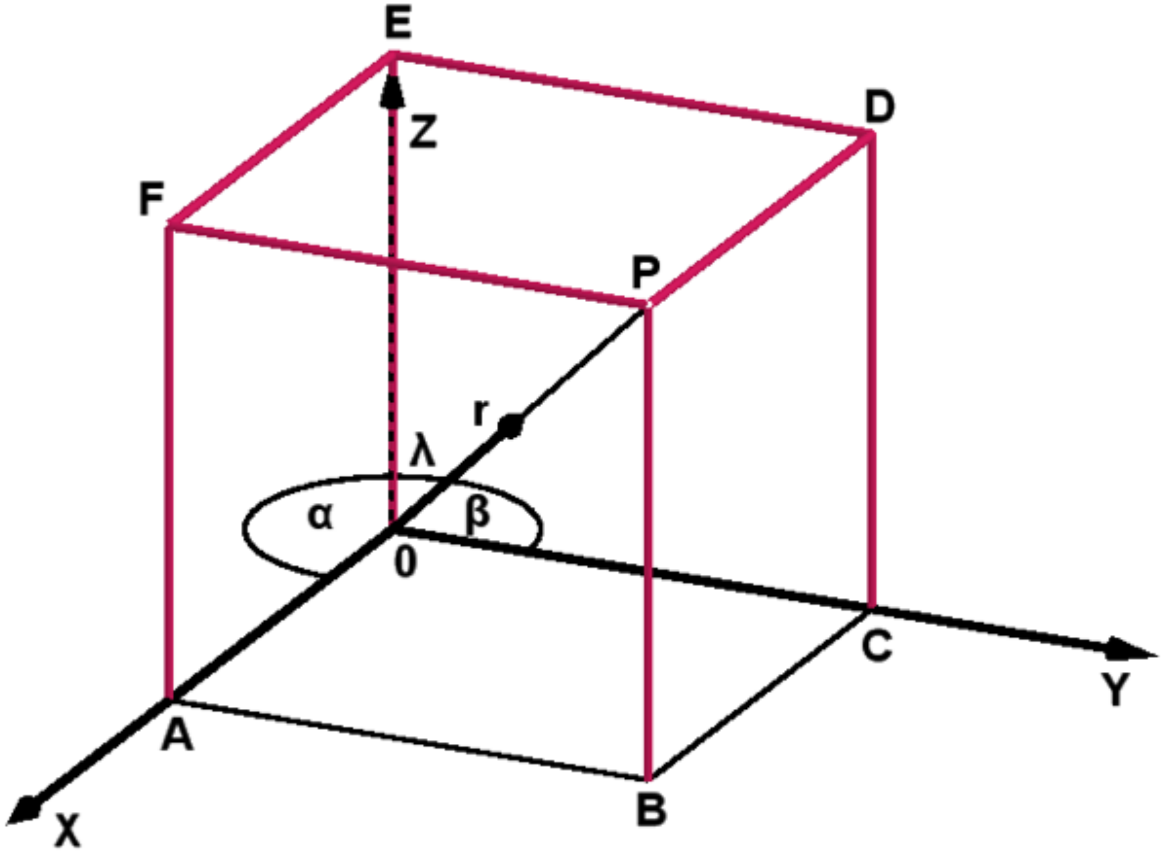


# Exercise

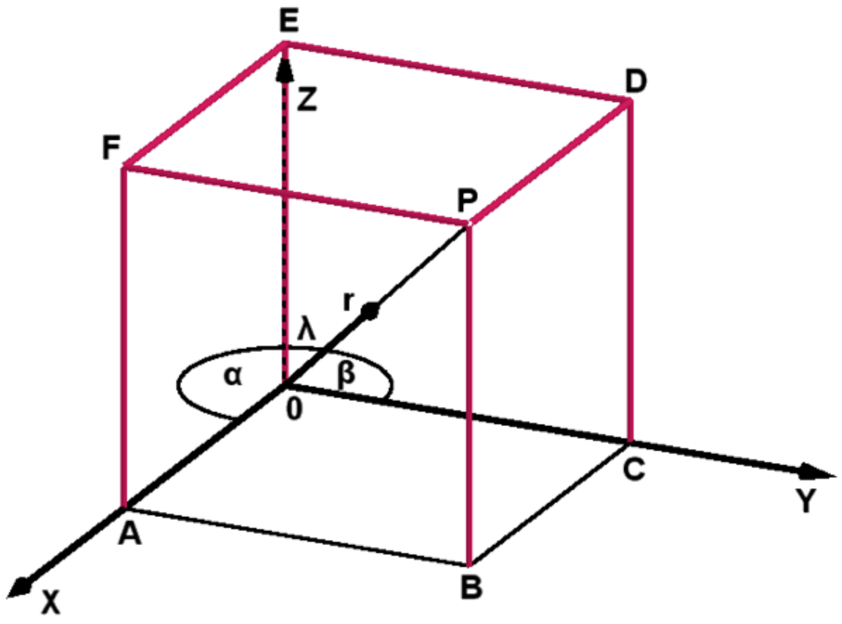
# Exercise

# Vectors in space (3-D)

In the 3-D space, we have *x*-axis, *y*-axis and *z*-axis



# DIRECTION COSINES



# Exercise

# Exercise

# EBS424: Vectors and Mechanics

## Unit 4: Multiplication of vectors, vector equation of a line and angles between two vectors

**Dr. Isaac Benning**
**Dept. of Maths & ICT Education**
**Faculty of Science and Technology Education**
**University of Cape Coast**

## Learning objectives

By the end of this unit, you should be able to:

(i) state and apply the dot/scaler product in solving problem.

(ii) find angle between two vectors.

(iii) determine cross product of given vectors.

(iv) determine the vector equation of a line.

(v) apply sine and cosine rules in solving problem.

# Watch the video lesson on dot and cross product

Click here 1

Click here 2

**You will tell the class what you learnt from the video.**

# Dot and cross product

- **Dot** and **cross product** make the development and computations of work and torque far simpler.

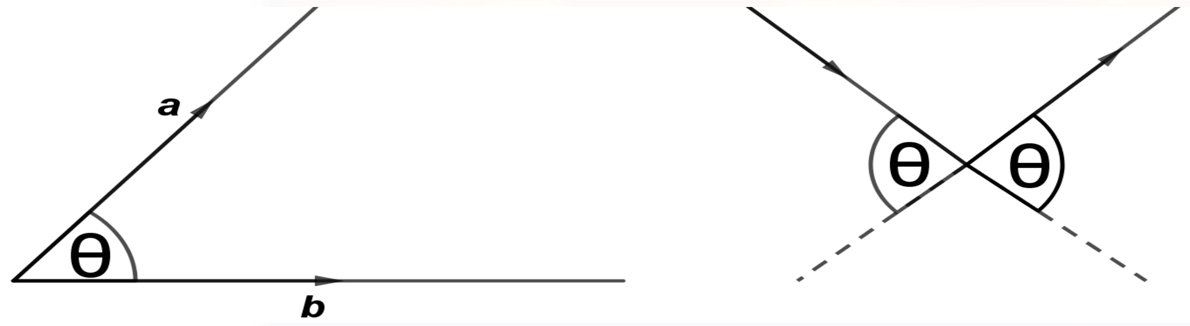
- Example
  - $W = F.d. = F.d.\cos\theta$

    ***F*** is the force applied, ***d*** is the displacement, ***θ*** is the angle between force and direction of motion.

  - $T = F \times r \times \sin\theta.$

    ***F*** = linear force. ***r*** = distance measured from the axis of rotation to where the application of linear force takes place.

# Dot/Scalar Product of Vectors




The scalar or dot product of two vectors is defined as the product of their magnitudes and the cosine of the angle between them.

$$a \cdot b = |a|\,|b| \cos\theta$$

# PROPERTIES OF DOT PRODUCTS

1. If the vectors *a* and *b* are parallel, then

$$\theta = 0° \text{ and } a \cdot b = |a||b|, \; because \cos 0° = 1$$

2. If the vectors *a* and *b* are perpendicular, then

$$a \cdot b = 0, \text{ because } \cos 90° = 0$$

3. The angle between the two vectors is given by

$$\theta = cos^{-1}\left(\frac{a \cdot b}{|a||b|}\right)$$

4. The unit vector *i* and *j* are perpendicular, so

$$i \cdot i = j \cdot j = 1 \text{ and } i \cdot j = j \cdot i = 0$$

# PROPERTIES OF DOT PRODUCTS

If $a, b\ and\ c$ are vectors in 3D and $k$ is a scalar, then

1. $[a] \cdot [a] = |a|^2$

2. $a \cdot b = b \cdot a$

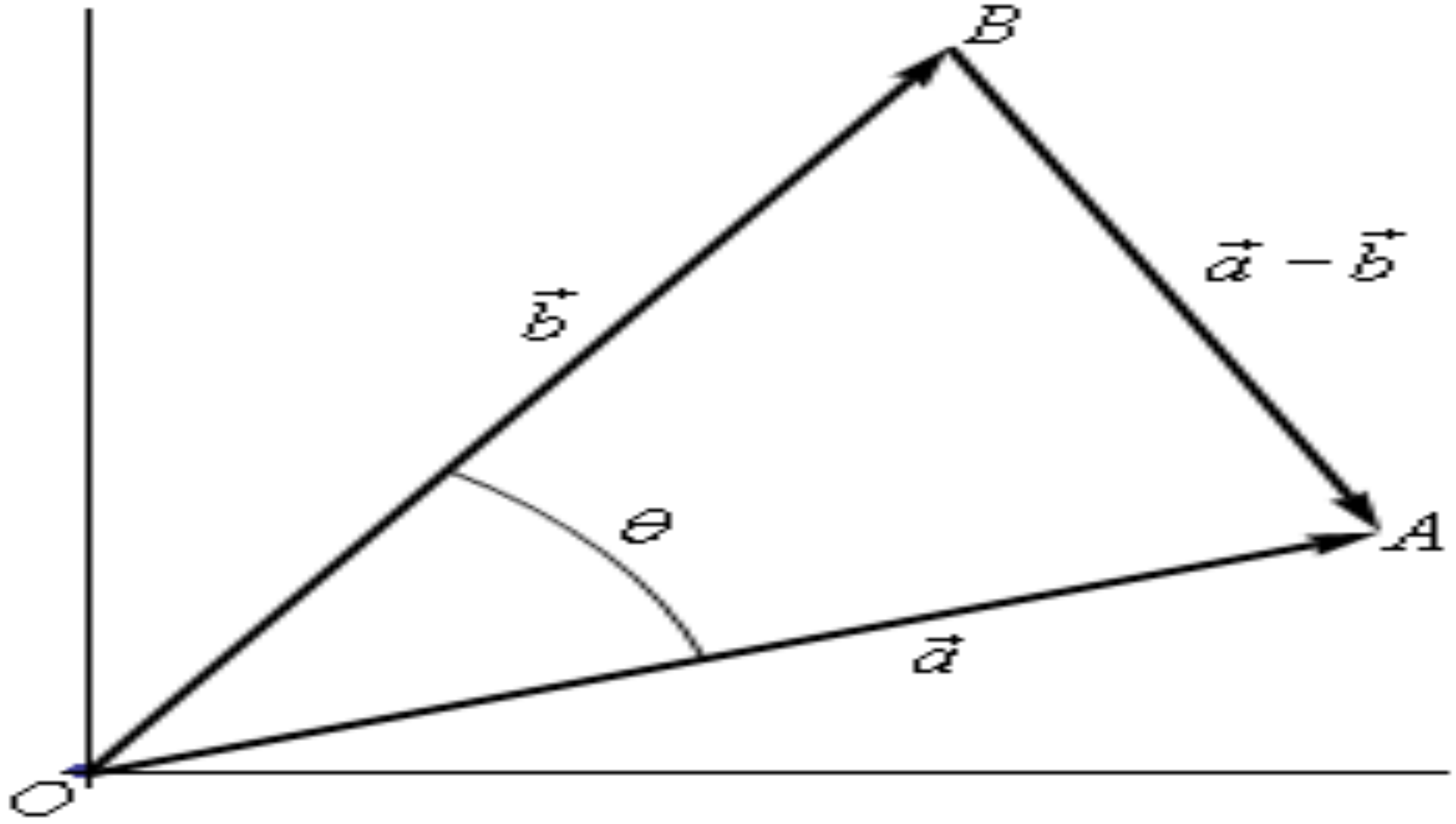
3. $a \cdot (b + c) = a \cdot b + a \cdot c$

4. $(ka) \cdot b = k(a \cdot b) = a \cdot (kb)$

5. $0 \cdot a = 0$

# Proof of the dot product

$$a \cdot b = |a|\,|b| \cos\theta$$



# Proof of the dot product

The three vectors above form the triangle *AOB* and note that the length of each side is nothing more than the magnitude of the vector forming that side.

The Law of Cosines tells us that,

$$\left\|\vec{a}-\vec{b}\right\|^2=\|\vec{a}\|^2+\left\|\vec{b}\right\|^2-2\,\|\vec{a}\|\,\left\|\vec{b}\right\|\cos\theta$$

Also using the properties of dot products we can write the left side as,

$$\begin{aligned}\left\|\vec{a}-\vec{b}\right\|^2&=\left(\vec{a}-\vec{b}\right)\centerdot\left(\vec{a}-\vec{b}\right)\\&=\vec{a}\centerdot\vec{a}-\vec{a}\centerdot\vec{b}-\vec{b}\centerdot\vec{a}+\vec{b}\centerdot\vec{b}\\&=\|\vec{a}\|^2-2\vec{a}\centerdot\vec{b}+\left\|\vec{b}\right\|^2\end{aligned}$$

Our original equation is then,

$$\begin{aligned}\left\|\vec{a}-\vec{b}\right\|^2&=\|\vec{a}\|^2+\left\|\vec{b}\right\|^2-2\,\|\vec{a}\|\,\left\|\vec{b}\right\|\cos\theta\\\|\vec{a}\|^2-2\vec{a}\centerdot\vec{b}+\left\|\vec{b}\right\|^2&=\|\vec{a}\|^2+\left\|\vec{b}\right\|^2-2\,\|\vec{a}\|\,\left\|\vec{b}\right\|\cos\theta\\-2\vec{a}\centerdot\vec{b}&=-2\,\|\vec{a}\|\,\left\|\vec{b}\right\|\cos\theta\\\vec{a}\centerdot\vec{b}&=\|\vec{a}\|\,\left\|\vec{b}\right\|\cos\theta\end{aligned}$$

# Scalar/dot product in terms of components

If vectors $a = x_1 i + y_1 j$ *and* $b = x_2 i + y_2 j$ , then;

$$a \cdot b = (x_1 i + y_1 j) \cdot (x_2 i + y_2 j)$$

$$a \cdot b = x_1 x_2\, i \cdot i + x_1 y_2 i \cdot j + y_1 x_2 j \cdot i + y_1 y_2\, j \cdot j$$

$$a \cdot b = x_1 x_2 (1) + x_1 y_2 (0) + y_1 x_2 (0) + y_1 y_2 (1)$$

$$a \cdot b = x_1 x_2 + y_1 y_2$$

# Example 1

If $a = 3i + 4j$ $and$ $b = 2i - 3j$, find $a \cdot b$

$$a \cdot b = (3i + 4j) \cdot (2i - 3j)$$

$$a \cdot b = 3\,(2) + 4\,(-3)$$

$$a \cdot b = 6 + (-12)$$

$$a \cdot b = -6$$

# Example 2

If $a = i + 2j - 3k$ and $b = 2j - k$, find $a \cdot b$

$$a \cdot b = (i + 2j - 3k) \cdot (0i + 2j - k)$$

$$a \cdot b = 1(0) + 2(2) + (-3)(-1)$$

$$a \cdot b = 0 + 4 + 3$$

$$a \cdot b = 7$$

# Angle between two vectors

- We can use the concept of the dot product,

$$a \cdot b = |a|\,|b| \cos\theta$$

to determine the angle between two vectors.

- If $\theta$ is the angle between the two vectors ***a*** and ***b*** then,

$$\theta = cos^{-1}\left(\frac{a \cdot b}{|a||b|}\right)$$

# Exercise

Find the angle between the vectors $a = 5i + 2j \;\; and \;\; b = 3i + 4j$

Solution

$$a \cdot b = (5i + 2j).(3i + 4j) = 15 + 8 = 23$$

$$|a| = \sqrt{25 + 4} = \sqrt{29}$$

$$|b| = \sqrt{9 + 16} = 5$$

$$a \cdot b = |a||b|cos\theta$$

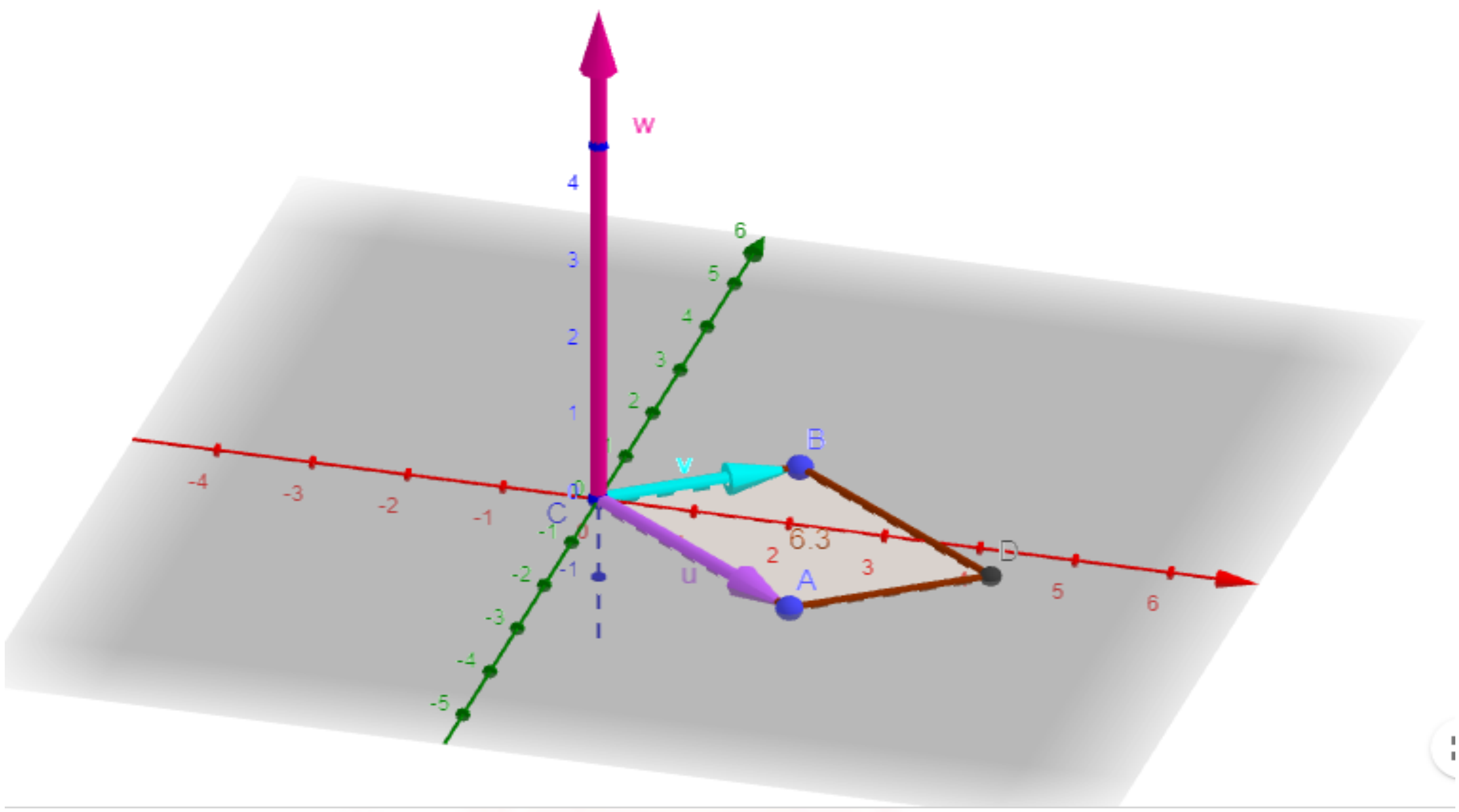
$$\theta = cos^{-1}\left(\frac{a \cdot b}{|a||b|}\right) = \left(\frac{23}{5\sqrt{29}}\right) = \cos^{-1} 0.8541986 = 31.33°$$

# Exercise

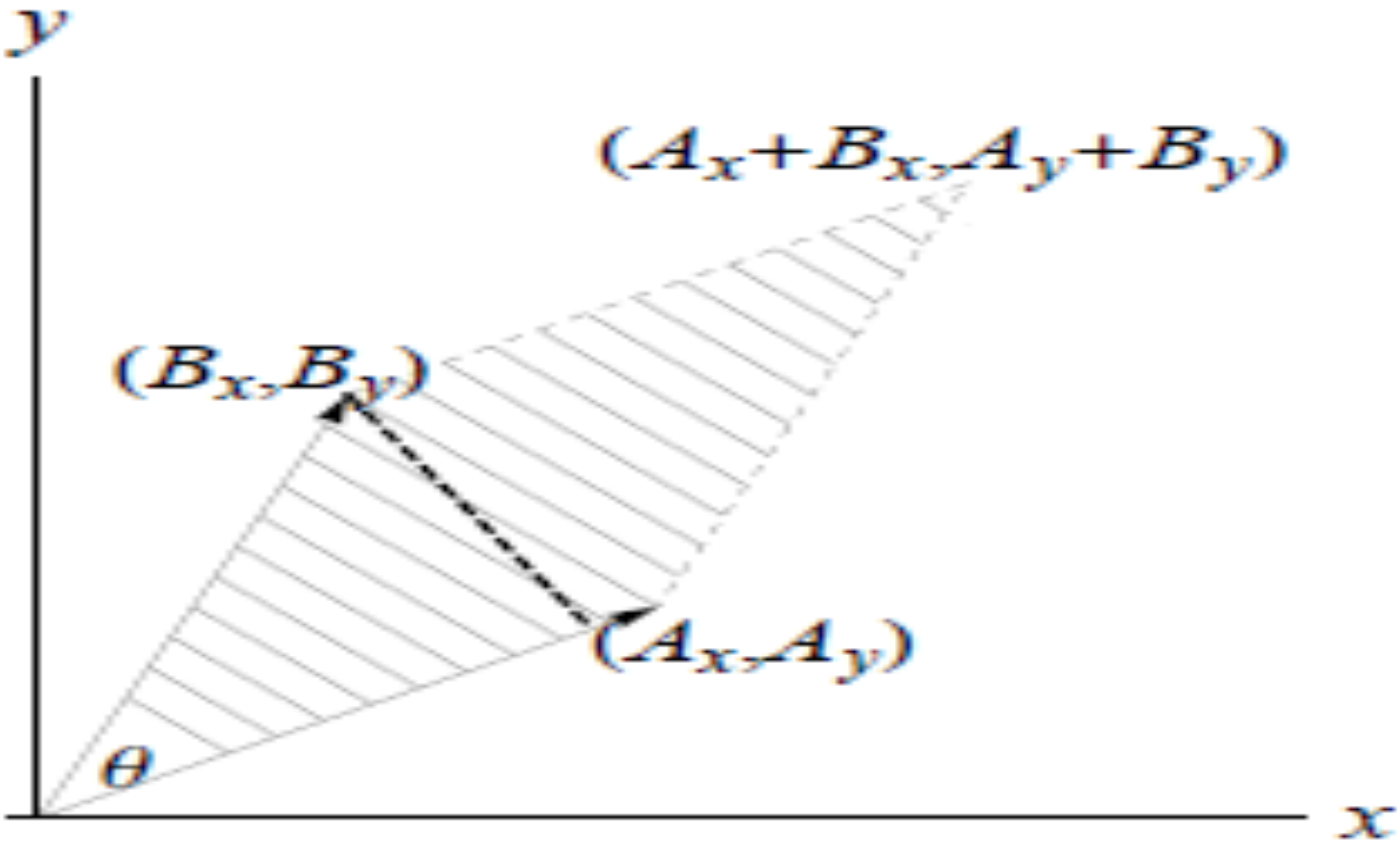
Find the angle between the vectors $a = 2i + 2j - k$ and $b = 5i - 3j + 2k$

# Geometry of cross product

Click here

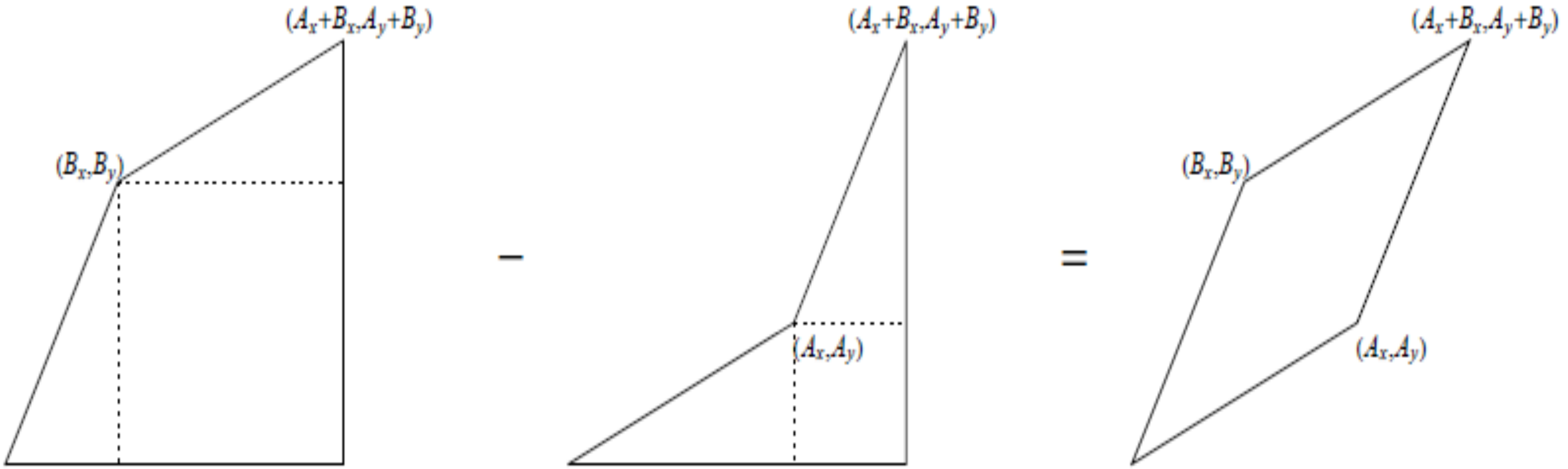


# Cross Product




Geometrical view of the cross product as the parallelogram area

# Cross Product

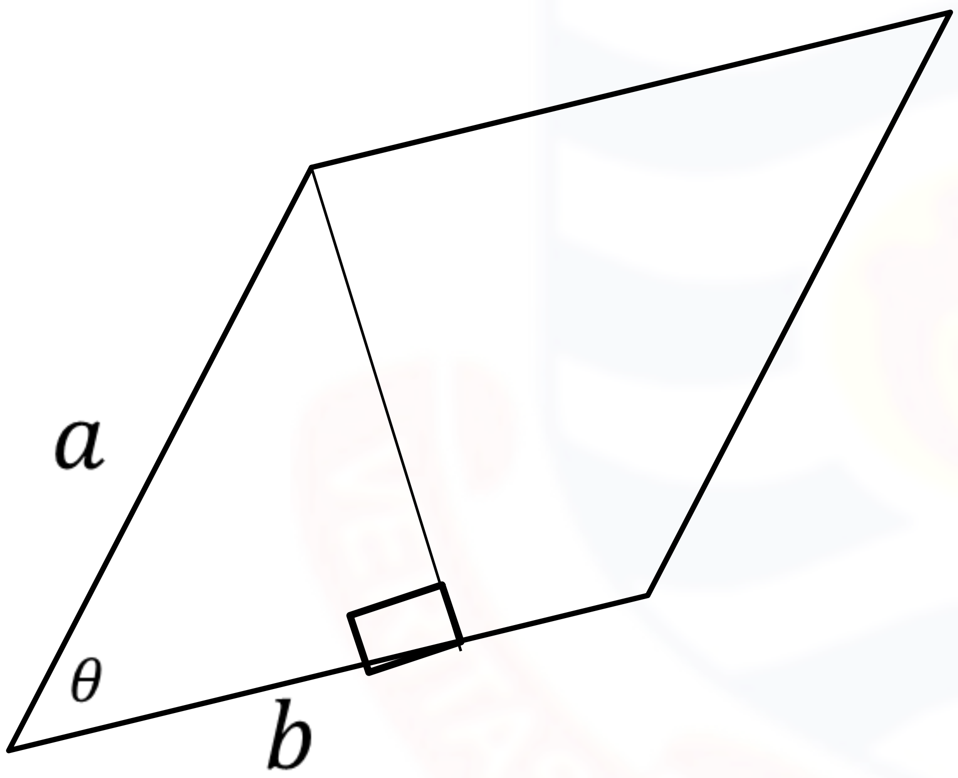



$$\frac{1}{2}B_xB_y+\frac{1}{2}A_xA_y+A_xB_y \quad - \quad \frac{1}{2}A_xA_y+\frac{1}{2}B_xB_y+B_xA_y \quad = \quad A_xB_y-B_xA_y$$

$$\text{area} = A_xB_y - B_xA_y$$

# Cross Product

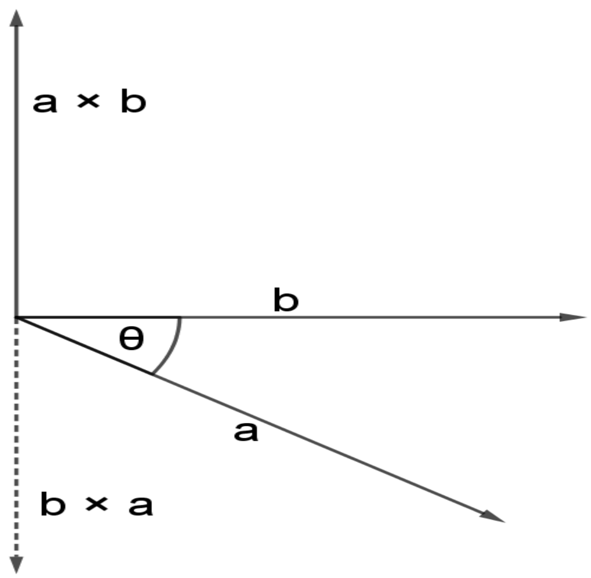
- The area of a parallelogram is

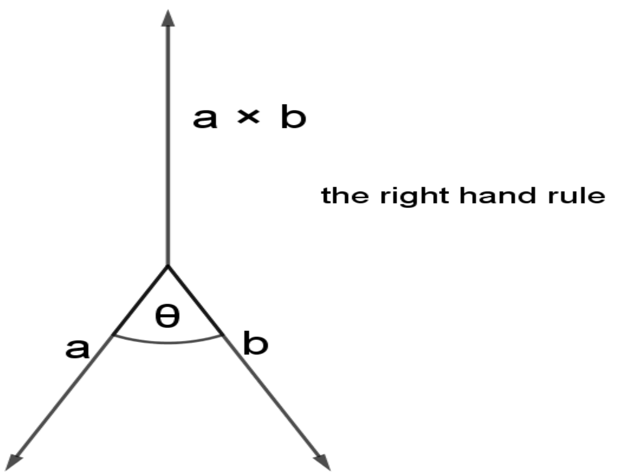



$$area = base \times height$$
$$= absin\theta$$

# THE CROSS PRODUCT

The cross product $\boldsymbol{a} \times \boldsymbol{b}$ of two vectors ***a*** and ***b***, unlike the dot product, is a vector. For this reason, it is also called the vector product.




The vector product of $(\mathbf{a} \times \mathbf{b})$ of ***a*** and ***b*** is defined as a vector having a magnitude $\boldsymbol{ab\ sin\theta}$ where θ is the angle between the two vectors. The product vector acts in a direction perpendicular to both ***a*** and ***b*** in such a sense that $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{a} \times \boldsymbol{b}$ form a right-handed set (rule) in that order.

$$\boldsymbol{a \times b = |a||b| sin\theta \ \ , where \ (0° \leq \theta \leq 180°)}$$

# Properties of vector product

1. $b \times a$ reverses the direction of rotation and the product vector would now act downward. That is $b \times a = -(a \times b)$.

2. If $\theta = 0°,\ then\ |a \times b| = 0$

3. If $\theta = 90°,\ then\ |a \times b| = ab$

# Properties of vector product

If $a, b$ $and$ $c$ are vectors, and k is a scalar, then

1. $a \times b = -b \times a$
2. $(ka) \times b = k(a \times b) = a \times (kb)$
3. $a \times (b + c) = a \times b + a \times c$
4. $(a + b) \times c = a \times c + b \times c$
5. $a \cdot (b \times c) = (a \times b) \cdot c$
6. $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$
7. The length of a cross product $a \times b$ is equal to the area of the parallelogram determined by $a$ and $b$

# Proof of the Vectors

4. If $a$ and $b$ are given in terms of the unit vectors $i, j$ $and$ $k$

$$a = a_1 i + a_2 j + a_3 k \quad and \quad b = b_1 i + b_2 j + b_3 k \quad then$$

$$a \times b = (a_1 i + a_2 j + a_3 k) \times (b_1 i + b_2 j + b_3 k)$$

$$a \times b = a_1 b_1\, i \times i + a_1 b_2\, i \times j + a_1 b_3 i \times j + a_2 b_1 i \times j + a_2 b_2 j \times j + a_2 b_3 j \times k + a_3 b_1 i \times k + a_3 b_2 j \times k + a_3 b_3 k \times k$$

……………………...

……………………

……………………... Complete the proof.

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \begin{matrix} unit\ vector \\ coefficient\ of\ a \\ coefficient\ of\ b \end{matrix}$$

$$\therefore a \times b = i \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - j \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

# 3D Dot and Cross Product

Click here to visualise the 3D dot and cross product

# Example

Find the vector product of $\underline{p}$ and $\underline{q}$ where $\underline{p} = 3i - 4j + 2k \quad and \; \underline{q} = 2i + 5j - k$

$$\underline{p} \times \underline{q} = \begin{vmatrix} i & j & k \\ 3 & -4 & 2 \\ 2 & 5 & -1 \end{vmatrix}$$

$$\underline{p} \times \underline{q} = \begin{vmatrix} -4 & 2 \\ 5 & -1 \end{vmatrix} i - \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} j + \begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix} k$$

$$= [(-4)(-1) - 2(-5)]i - [3(-1) - 2(2)]j + [3(5) - (2)(-4)]k$$

$$= (4 - 10)i - (-3 - 4)j + (15 + 8)k$$

$$= -6i + 7j + 23k$$

# Exercise

1. Find the vector product of $\underline{a}$ and $\underline{b}$ where

$$\underline{a} = i + 3j + 4k \quad and \quad \underline{b} = 2i + 7j - 5k$$

**Ans:** $\boldsymbol{-43i + 13j + k}$

# Application 1

Find the area of the triangle with vertices $P = (1, 2, 3)$, $Q = (4, 2, 6)$ and $R = (5, 3, 7)$.

Two sides of the triangle are formed by the vectors $\overrightarrow{PQ} = \langle 4-1, 2-2, 6-3 \rangle = \langle 3, 0, 3 \rangle$ and $\overrightarrow{PR} = \langle 5-1, 3-2, 7-3 \rangle = \langle 4, 1, 4 \rangle$. •

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 3 \\ 4 & 1 & 4 \end{vmatrix} = \mathbf{i} \begin{vmatrix} 0 & 3 \\ 1 & 4 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 3 & 3 \\ 4 & 4 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 3 & 0 \\ 4 & 1 \end{vmatrix}$$

$$= [0(4) - 3(1)]\mathbf{i} - [3(4) - 3(4)]\mathbf{j} + [3(1) - 0(4)]\mathbf{k}$$

$$= -3\mathbf{i} - 0\mathbf{j} + 3\mathbf{k} = \langle -3, 0, 3 \rangle \bullet$$

$$[\text{Area of the triangle}] = \tfrac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}| = \tfrac{1}{2}|\langle -3, 0, 3 \rangle| = \tfrac{1}{2}\sqrt{3^2 + 0^2 + 3^2}$$

$$= \tfrac{1}{2}\sqrt{18} = \tfrac{3}{2}\sqrt{2}.$$

# Application 2

Calculate $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ for $\mathbf{u} = \langle 3, 3, -1 \rangle$, $\mathbf{v} = \langle 4, 6, 5 \rangle$, and $\mathbf{w} = \langle 2, 2, -1 \rangle$.

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \langle 3, 3, -1 \rangle \cdot (\langle 4, 6, 5 \rangle \times \langle 2, 2, -1 \rangle)$$

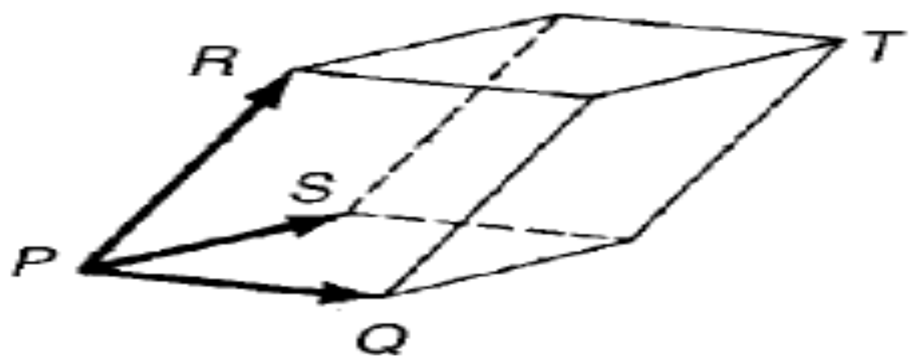
$$= \begin{vmatrix} 3 & 3 & -1 \\ 4 & 6 & 5 \\ 2 & 2 & -1 \end{vmatrix} = 3 \begin{vmatrix} 6 & 5 \\ 2 & -1 \end{vmatrix} - 3 \begin{vmatrix} 4 & 5 \\ 2 & -1 \end{vmatrix} + (-1) \begin{vmatrix} 4 & 6 \\ 2 & 2 \end{vmatrix}$$

$$= 3[6(-1) - 5(2)] - 3[4(-1) - 5(2)] - [4(2) - 6(2)]$$

$$= 3(-16) - 3(-14) - (-4) = -48 + 42 + 4 = -2$$

# Application 3

What is the volume of the parallelepiped with vertex $P = (1, 1, 1)$ and adjacent vertices $Q = (4, 4, 0)$, $R = (5, 7, 6)$, and $S = (3, 3, 0)$?




Adjacent sides of the parallelepiped are formed by

$\overrightarrow{PQ} = \langle 4-1, 4-1, 0-1 \rangle = \langle 3, 3, -1 \rangle$, $\overrightarrow{PR} = \langle 5-1, 7-1, 6-1 \rangle = \langle 4, 6, 5 \rangle$ and

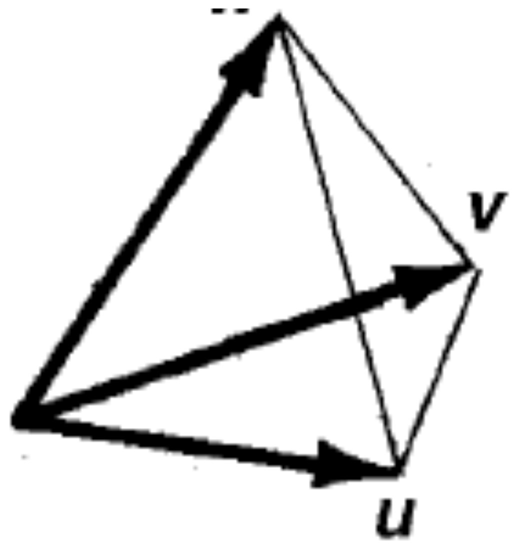
$\overrightarrow{PS} = \langle 3-1, 3-1, 0-1 \rangle = \langle 2, 2, -1 \rangle$. •

The triple product $\overrightarrow{PQ} \cdot (\overrightarrow{PR} \times \overrightarrow{PS})$ = Volume of the parallelepiped

[Volume of the parallelepiped] $= |-2| = 2$

# Application 4

The vectors $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$ with their bases at the origin form three edges of a tetrahedron. What is its volume?




$$\mathbf{i} \cdot (\mathbf{j} \times \mathbf{k}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \quad \bullet \quad [\text{Volume}] = \tfrac{1}{6}$$

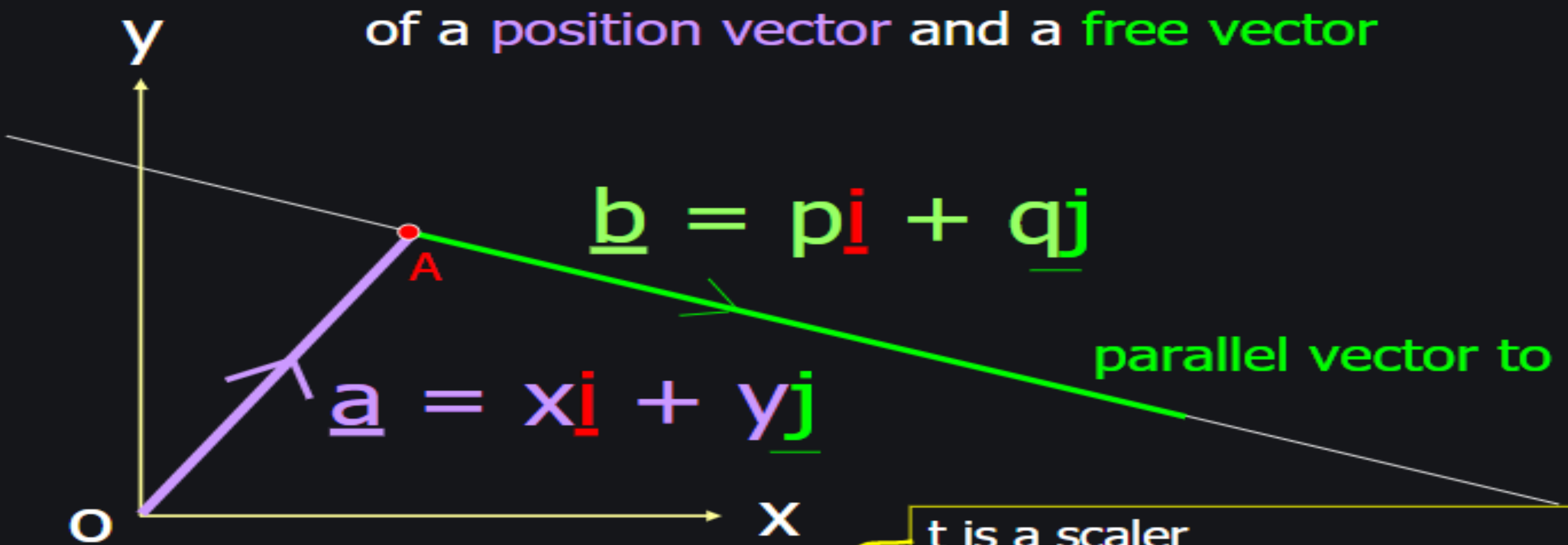
# Exercise

1. Find the area of the triangle with vertices $P(1, 4, 6), Q(-2, 5, \ -1) \ \ and \ R(1, -1, 1)$.

**Ans:** $\frac{5\sqrt{82}}{2}$

# The vector equation of a line

A line can be identified by a linear combination of a position vector and a free vector




t is a scaler
- it can be any number,
since we only need a parallel vector

E.g. $\underline{a} + t\underline{b}$

$$= (x\underline{i} + y\underline{j}) + t(p\underline{i} + q\underline{j})$$

# Vector Equation of a Line

1. Vector equation of the straight line passing through origin and parallel to $\boldsymbol{b}$ is given by

$$\boldsymbol{r} = t\boldsymbol{b}$$

2. Vector equation of the straight line passing through $\boldsymbol{a}$ and parallel to $\boldsymbol{b}$ is given by

$$\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$$

3. Vector equation of the straight line passing through $\boldsymbol{a}$ and $\boldsymbol{b}$ is given by

$$\boldsymbol{r} = \boldsymbol{a} + t(\boldsymbol{b} - \boldsymbol{a})$$

# Exercise

Find the vector equation of a line passes through the points A(2, 5) and B(-4, 1). Hence (a) write the parametric vector equation and (b) symmetric vector equation.

**Solution**

Let ***r*** be the vector equation of the line,

$$\boldsymbol{r} = \boldsymbol{a} + t(\boldsymbol{b} - \boldsymbol{a})$$

$$\boldsymbol{b} - \boldsymbol{a} = (-4i + j) - (2i + 5j) = -6\mathrm{i} - 4\mathrm{j}$$

$$\boldsymbol{r} = (2i + 5j) + t(-6i - 4j)$$

(a) The vector equation *r* can also be written in **parametric** form as

$$x = 2 - 6t \text{ and } y = 5 - 4t$$

# Exercise

(b) We can solve for $t$ from the parametric equation,

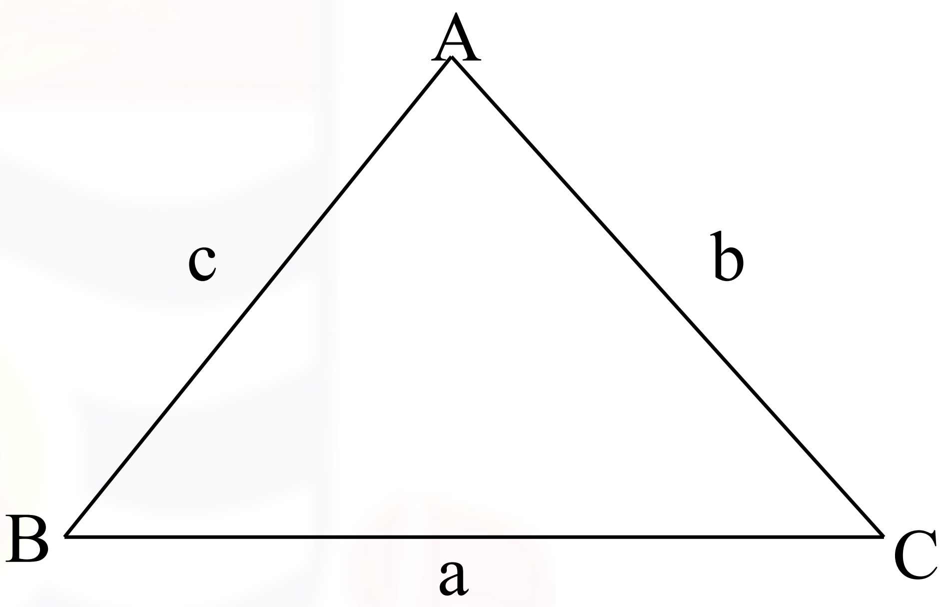
$$x = 2 - 6t \text{ and } y = 5 - 4t$$

$$t = \frac{x-2}{-6} = \frac{y-5}{-4}$$

$\frac{x-2}{-6} = \frac{y-5}{-4}$ is called the **symmetric form** of the equation

# The sine rule

$\Delta ABC$ has sides $AB = c$, $AC = b$ and $BC = a$. A, B and C are the angles at the vertices of the triangle. The sine rules enable us to calculate sides and angles in the same triangles where there is no a right angle.




**The sine rule is given by**

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

or

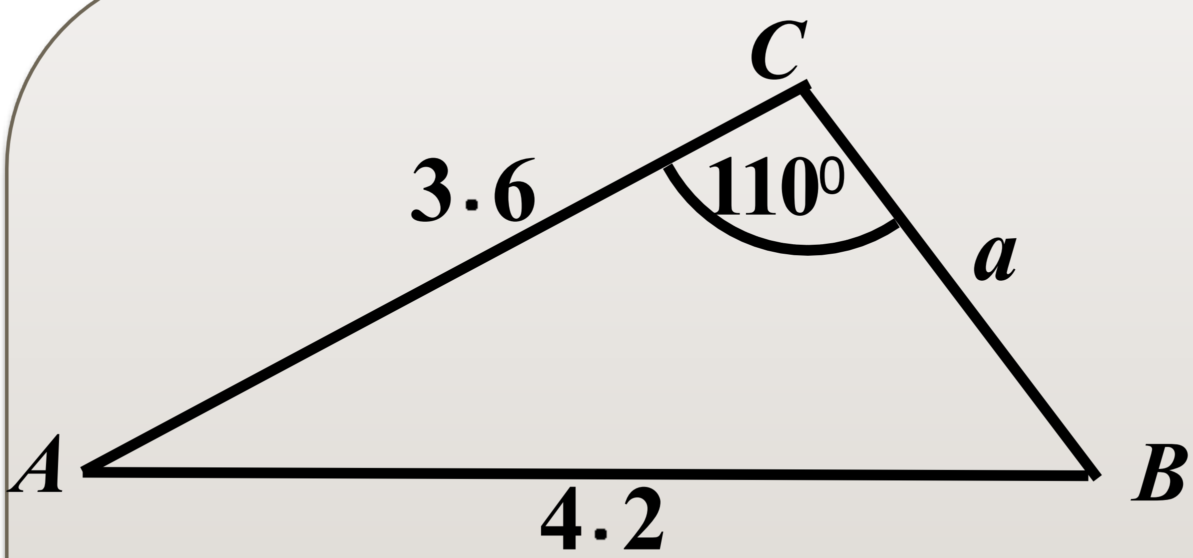
$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

The Sine Rule is used to solve any problems involving triangles when at least either of the following is known:

1. two angles and a side
2. two sides and an angle opposite a given side (a non-included angle)

# Exercise

1. In triangle $ABC$, $b = 3.6$ cm, $c = 4.2$ cm and angle $C = 110^\circ$. Find the size of angles $A$ and $B$.




$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\sin B = \frac{3.6 \sin 110}{4.2}$$

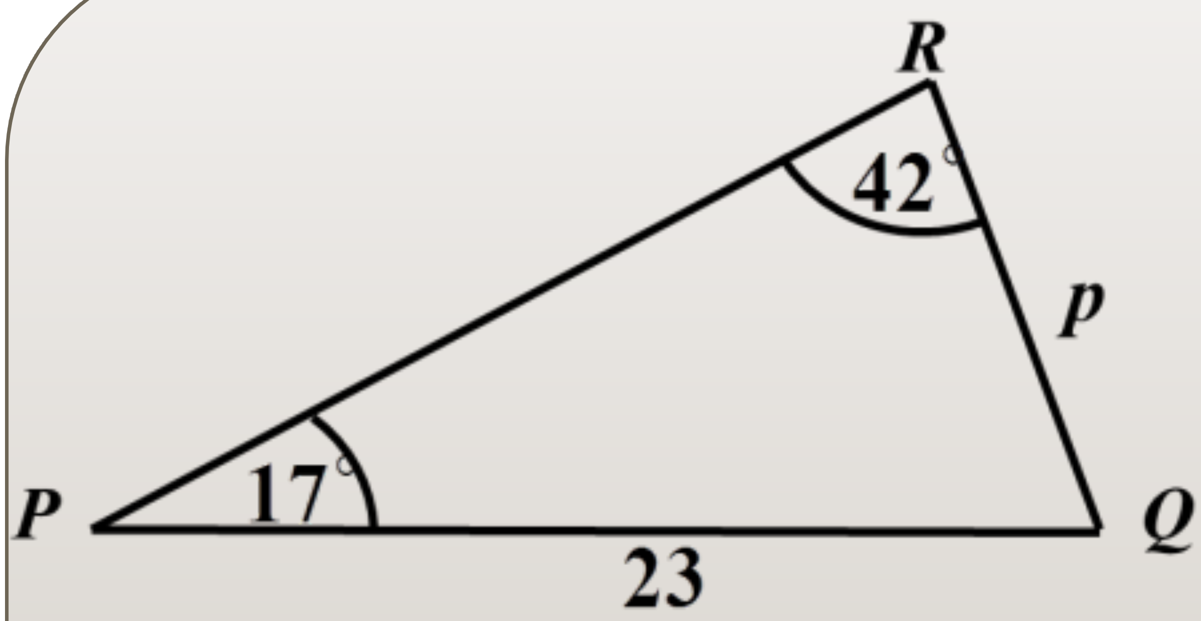
$$B = 53.7^0$$

*Now find out angle A ?*

$$A = 16.3^0$$

# Exercise

2. In triangle *PQR*, *PQ* = 23 cm, angle *R* = 42˚ and angle *P* = 17˚. Find the size of side *QR*.



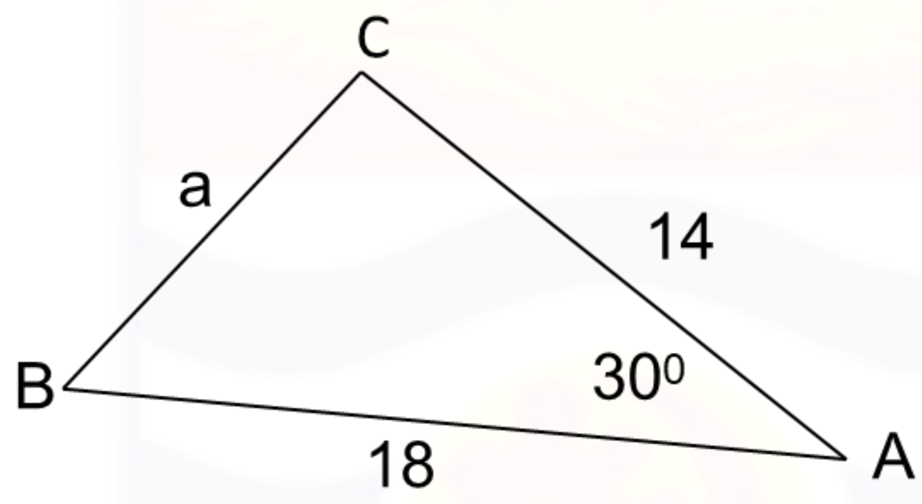

$$\frac{p}{\sin P} = \frac{r}{\sin R}$$

$$p = \frac{23\sin 17^\circ}{42^\circ}$$

$$p = 10.0cm$$

# The Cosine Rule

Sometimes the sine rule is not enough to help us solve for a non right angled triangle. Example is the case below.




The sine rule only provided the Following

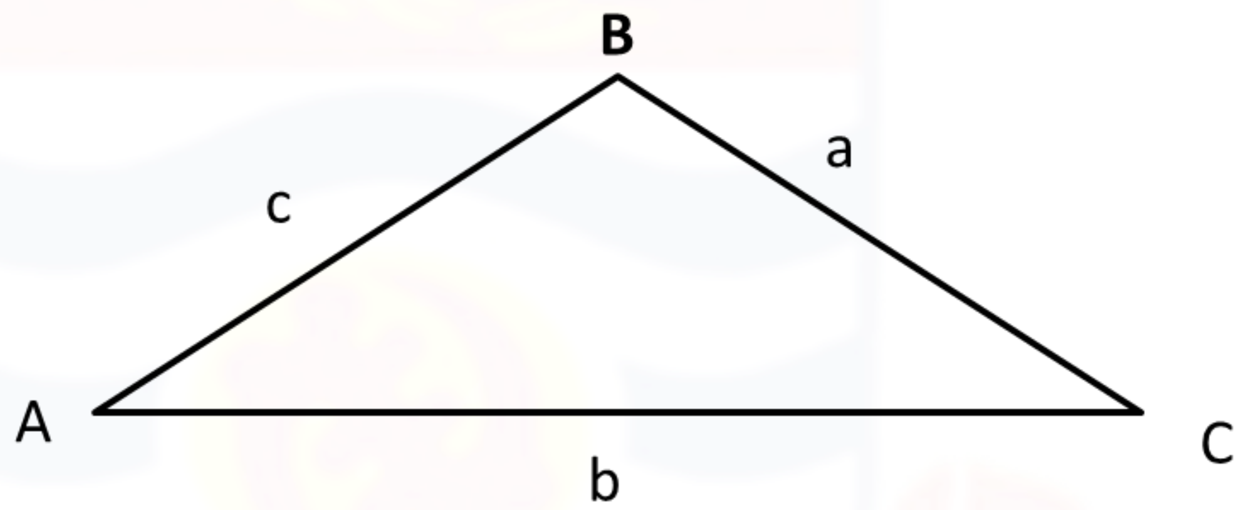
$$\frac{a}{sin30^0} = \frac{14}{sinB^0} = \frac{18}{sinC}$$

For this reason we need another useful rule, known as the **COSINE RULE**. The Cosine Rule maybe used when:

# The Cosine Rule

**The Cosine Rule maybe used when:**

1. Three sides are given.
2. Two sides and an included angle are given.




## The cosine Rule is given by:

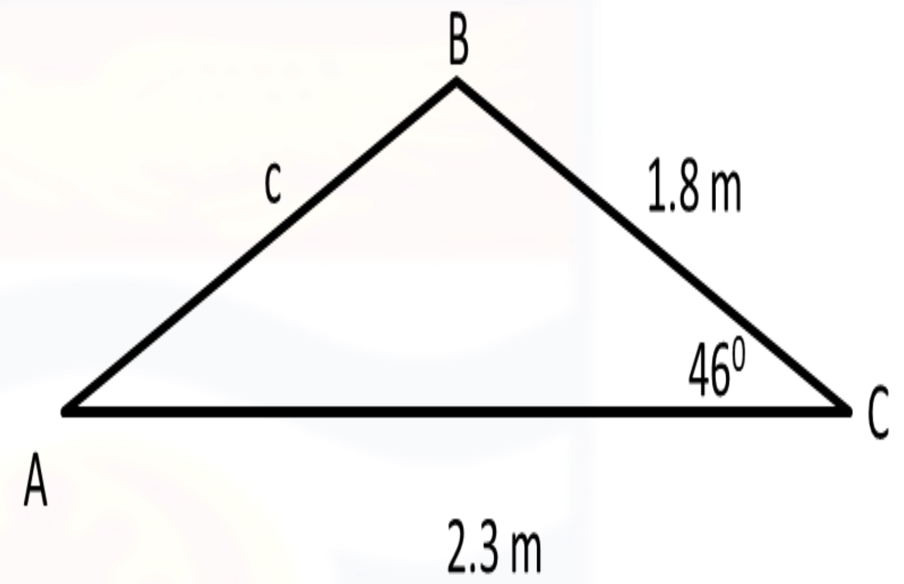
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B \quad a^2$$

$$c^2 = + b^2 - 2ab \cos C$$

# Exercise

In Triangle ABC, length of a = 1.8m, length of b = 2.3m and an angle of C = 46˚. Find the length of c.




Using the cosine rule,

$$c^2 = a^2 + b^2 - 2abcosC$$

$$c^2 = 1.8^2 + 2.3^2 - 2(1.8)(2.3)cos46^0$$

$$c^2 = \sqrt{2.778}$$

$$c = 1.667$$

# Some Application Questions

1. Two ships **A** and **B** leaves a port simultaneously. **A** steams at 10 $kmh^{-1}$ on a bearing of $160^0$ and **B** steams on a bearing of $215^0$. Just after one hour the bearing of **B** from **A** is $260^0$. Find the speed of **B**, correct to two significant figures.

**Ans: 14 $kmh^{-1}$**

# Some Application Questions

2. A ship steams from Port **P** for a distance of 15 km on a bearing of $070^0$ to port **Q**. It then steams from port **Q** to **R** a distance of 20 km on a bearing of $125^0$. Find the distance and bearing of port **R** from port **P**. Leave the answer in the nearest whole number.

**Ans: (29.6 km, $108^0$)**

# Some Application Questions

3. A boat is rowed with speed 8 $kmh^{-1}$ straight across a river, which is flowing at speed 6 $kmh^{-1}$. Find the resultant velocity of the boat. If the breadth of the river is 100 m, find how far down the river will the boat reached the opposite bank.

**Ans:**

a. The resultant velocity is 10 $kmh^{-1}$ on the direction of $53.13^0$ with the bank of the river.

b. The boat will be carried down stream a distance of 75 m

# Some Application Questions

4. Kwame and Yaro set out from school simultaneously. Kwame goes due north, and Yaro runs on a bearing of $047^0$. When Yaro has gone 550 m, the boys are 500 m apart. How far is Kwame from the school?

**Ans: 672 m**

# EBS424: Vectors and Mechanics

# Unit 6: Mechanics 2-Dynamics

**Dr. Isaac Benning**
**Dept. of Maths & ICT Education**
**Faculty of Science and Technology Education**
**University of Cape Coast**

# Learning objectives

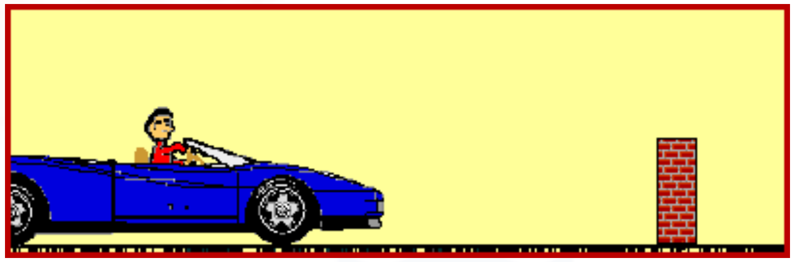
By the end of this unit, you should be able to:

- state and use Newton's law of motion
- State and apply the equation of motion (under uniform acceleration)
- solve problems involving motion under gravity.
- Solve problems related to projectiles
- find the force up an inclined plane.
- State and apply the principles of conservation of linear momentum.

# Newton’s First law of motion

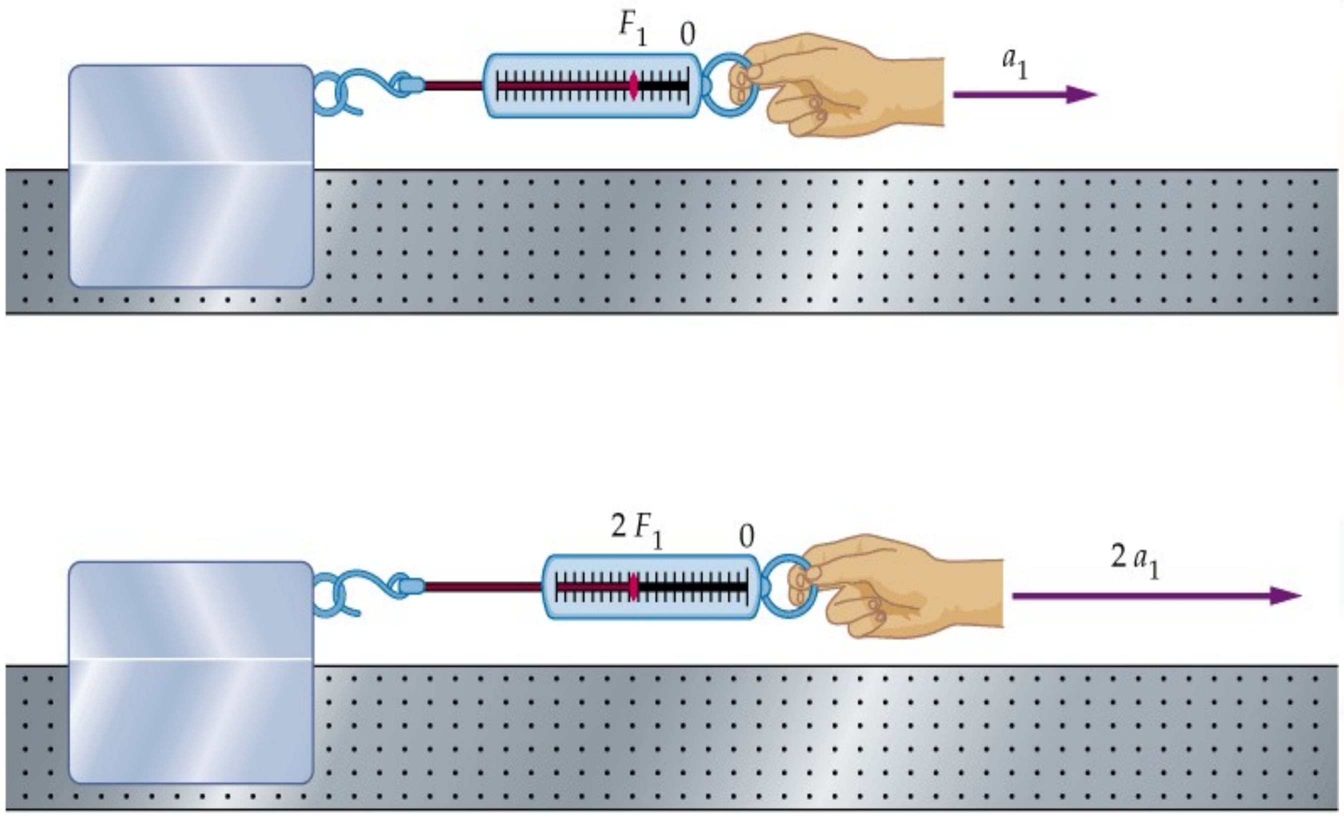
**First law: An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force.**

- An object at rest remains at rest as long as no net force acts on it.
- An object moving with constant velocity continues to move with the same speed and in the same direction (the same velocity) as long as no net force acts on it.
- *“Keep on doing what it is doing”*
- When forces are balanced, the acceleration of the object is zero.
  - Object at rest: $v = 0$ and $a = 0$
  - Object in motion: $v \neq 0$ and $a = 0$



# Newton's second law of motion

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass

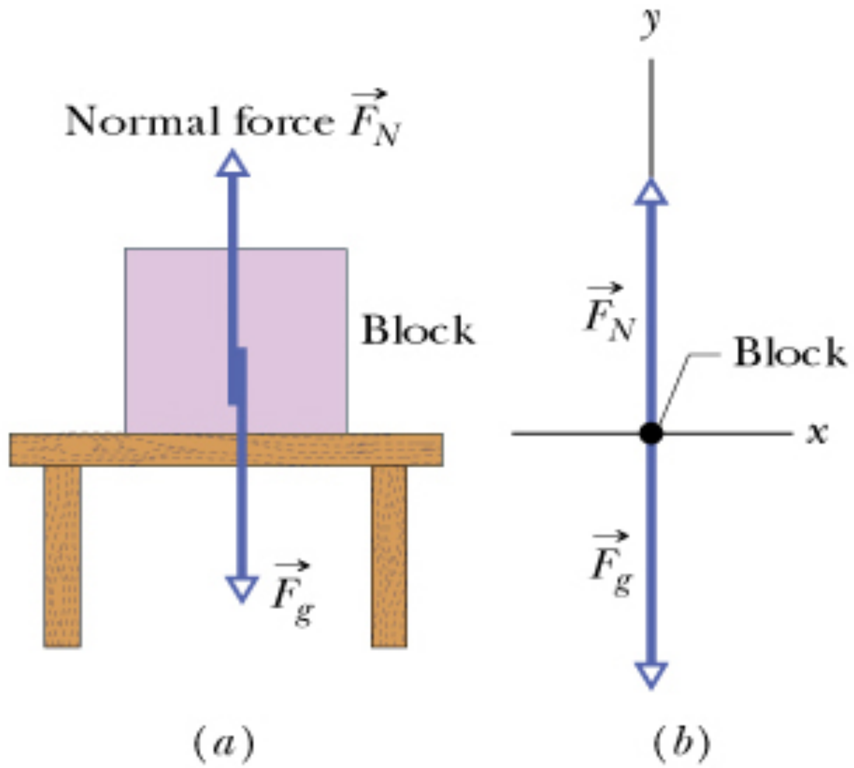



$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}$$

# Normal force

- Force from a solid surface which keeps object from falling through
- Direction: always perpendicular to the surface
- Magnitude: depends on situation




$$N - F_g = ma_y$$

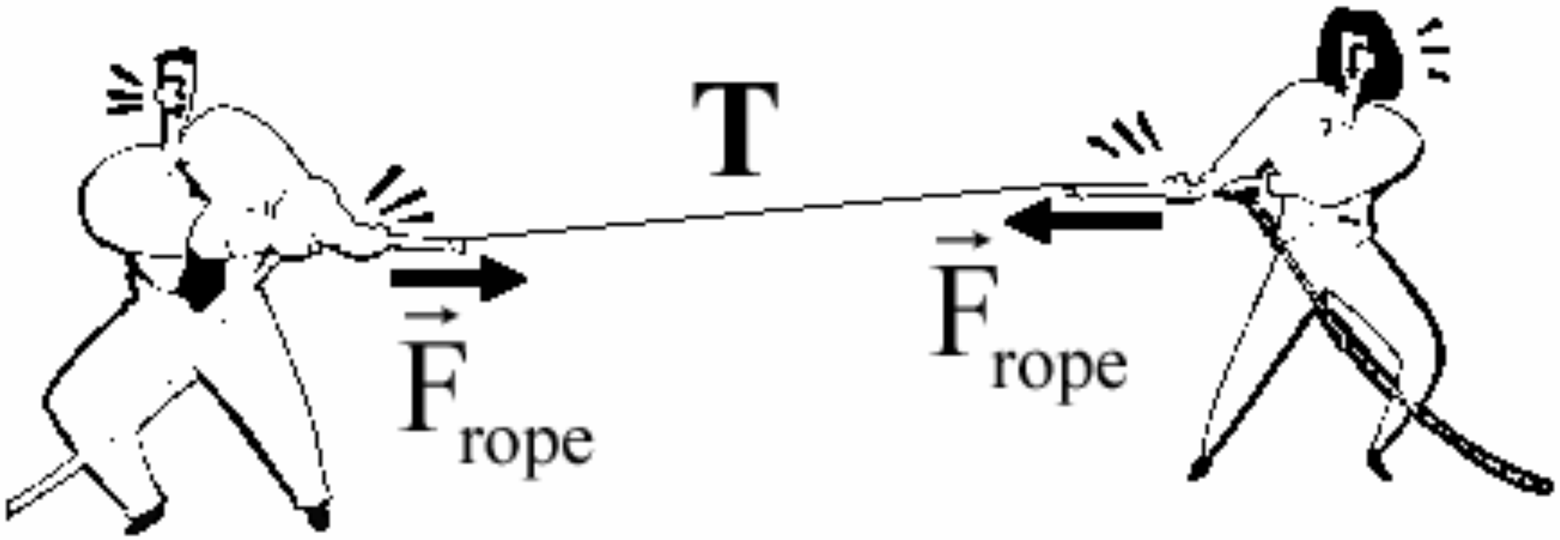
$$N - mg = ma_y$$

$$N = mg$$

# Newton’s third law of motion

- To every action, there is an equal and opposite reaction.

- If object 1 and object 2 interact, the force exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force exerted by object 2 on object 1

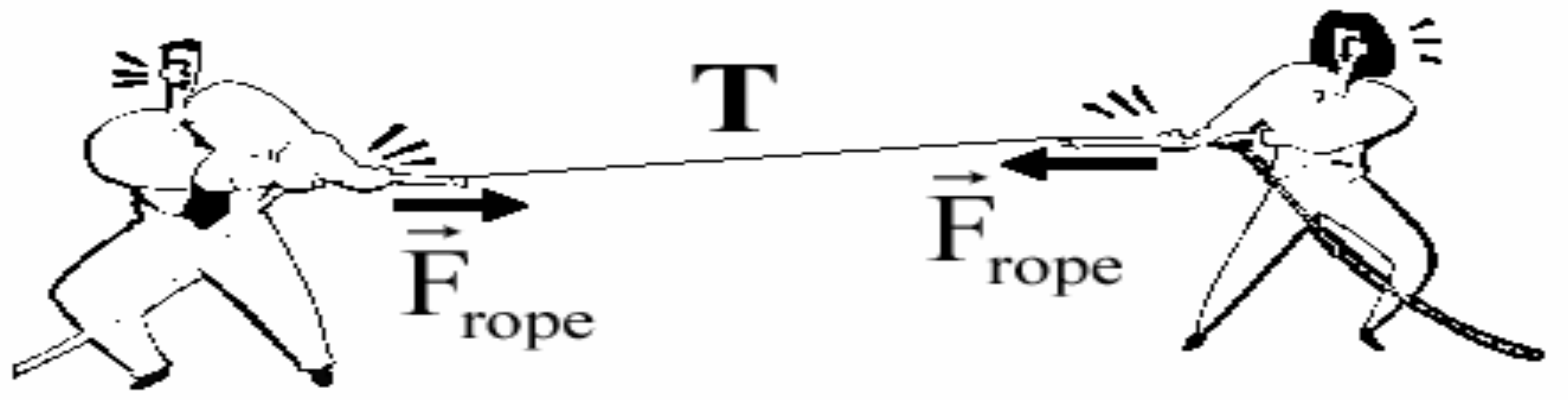



$$\left|\vec{F}_{\text{on A}}\right| = T = \left|\vec{F}_{\text{on B}}\right|$$

$$\vec{F}_{on\ A} = -\vec{F}_{on\ B}$$

- Equivalent to saying a single isolated force cannot exist

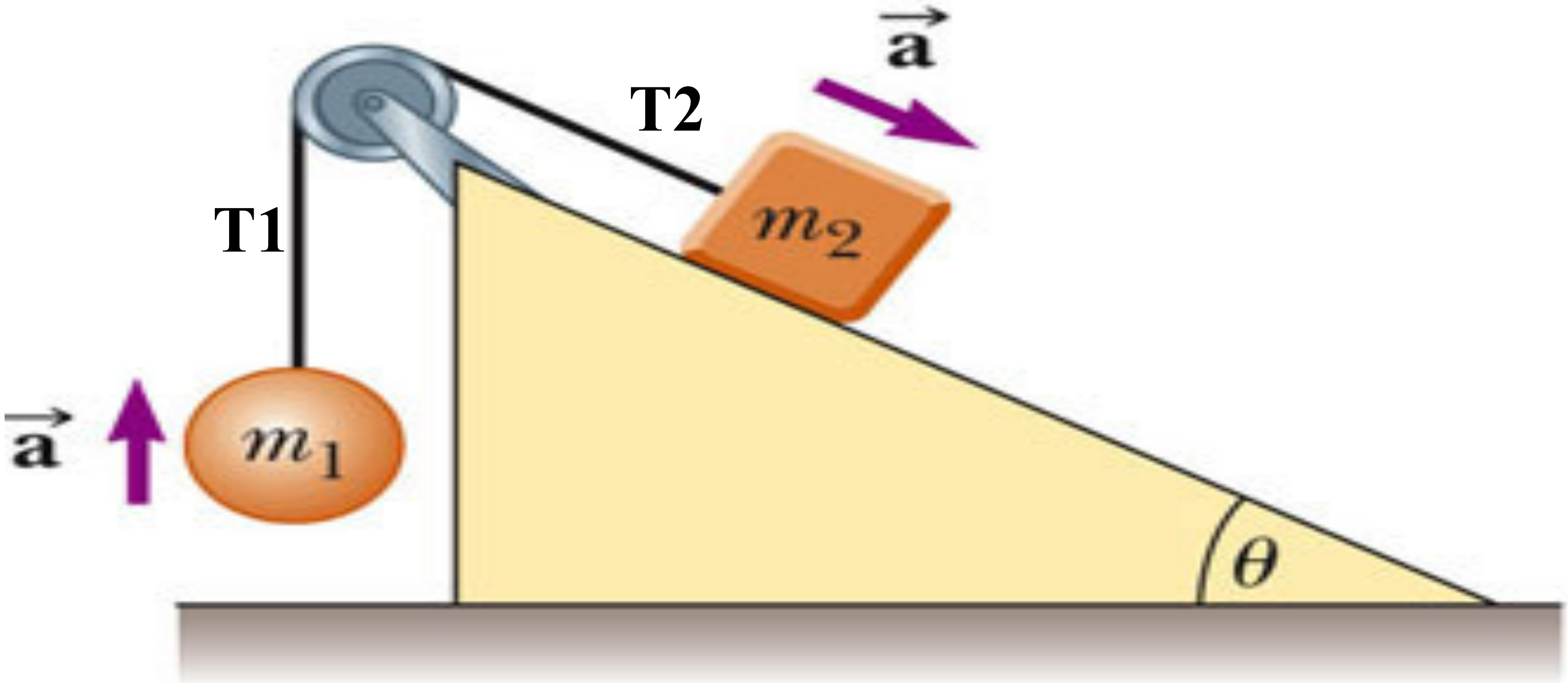
# Tension Force




$$\left|\vec{F}_{\text{on A}}\right| = T = \left|\vec{F}_{\text{on B}}\right|$$

- A taut rope exerts forces on whatever holds its ends
- Direction: always along the cord (rope, cable, string ……) and away from the object
- Magnitude: depend on situation

# Tension Force

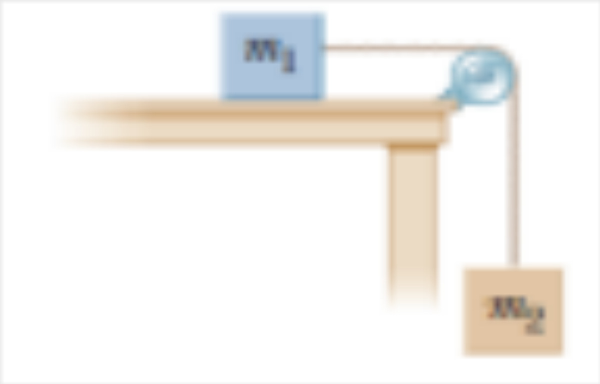


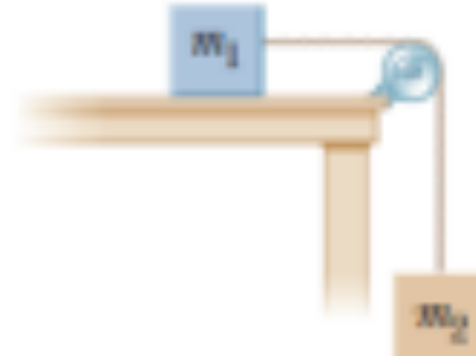

**T1 = T = T2**

# Worked example

An object of mass $m_1 = 3.80\text{-}kg$ placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging object of mass $m_2 = 7.20\text{-}kg$ as shown in the figure.

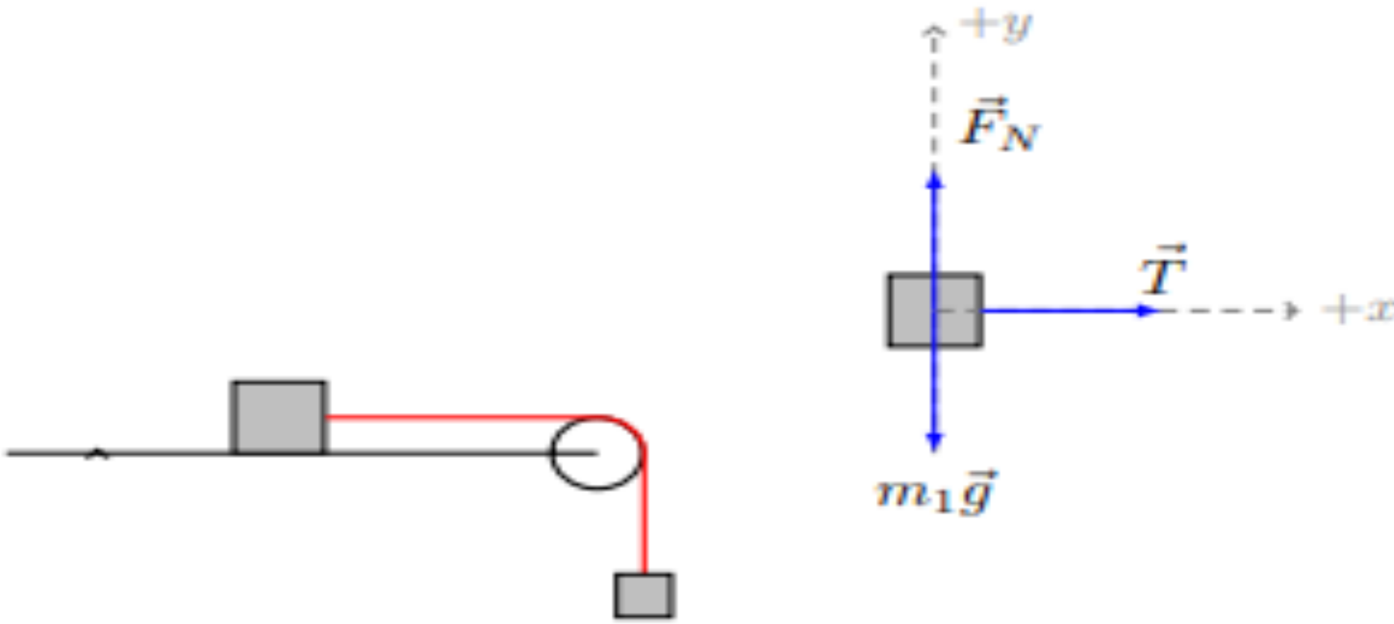
- a) Draw free-body diagrams of both objects.
- b)Find the magnitude of the acceleration of the objects.
- ) Find the tension in the string.






## a) Free body diagrams

- Object




## b)

- For object $m_1$

  $\sum \vec{F} = m_1\vec{g} + \vec{T} + \vec{F}_N = m_1\vec{a_1}$ ; $\quad \vec{a}_1 = a_1\hat{i}$ (horizontal motion)

  Prejection on the x-axis should give:

  $0 + T + 0 = m_1 a_1 \qquad (1)$

- For object $m_2$

$$\sum \vec{F} = m_2\vec{g} + \vec{T'} = m_2\vec{a_2} \;; \qquad \vec{a}_2 = a_2\hat{j} \text{ (vertical motion)}$$

Prejection on the y-axis (+ sign downward only for convenience)should give:

$$m_2 g - T' = m_2 a_2 \qquad (2)$$

- the two objects are connected together and the magnitude of their accelerations is the same: $a_1 = a_2 = a$

the string tensions are also the same $T = T'$

We can derive :

- For object $m_2$

$$\sum \vec{F} = m_2\vec{g} + \vec{T'} = m_2\vec{a_2} \; ; \qquad \vec{a}_2 = a_2\hat{j} \text{ (vertical motion)}$$

Prejection on the y-axis (+ sign downward only for convenience)should give:

$$m_2 g - T' = m_2 a_2 \qquad (2)$$

- the two objects are connected together and the magnitude of their accelerations is the same: $a_1 = a_2 = a$

the string tensions are also the same $T = T'$

We can derive :

$$\begin{aligned} m_1 a &= T \\ &= m_2 g - m_2 a \end{aligned}$$

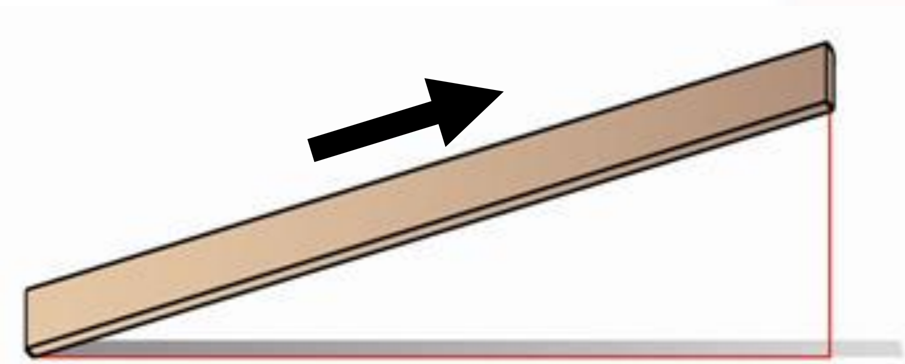
$$
\begin{aligned}
a &= \frac{m_2 g}{m_1 + m_2} \\
&= \frac{7.20 \times 9.81}{7.20 + 3.80} \\
&= 6.42 m/s^2
\end{aligned}
$$

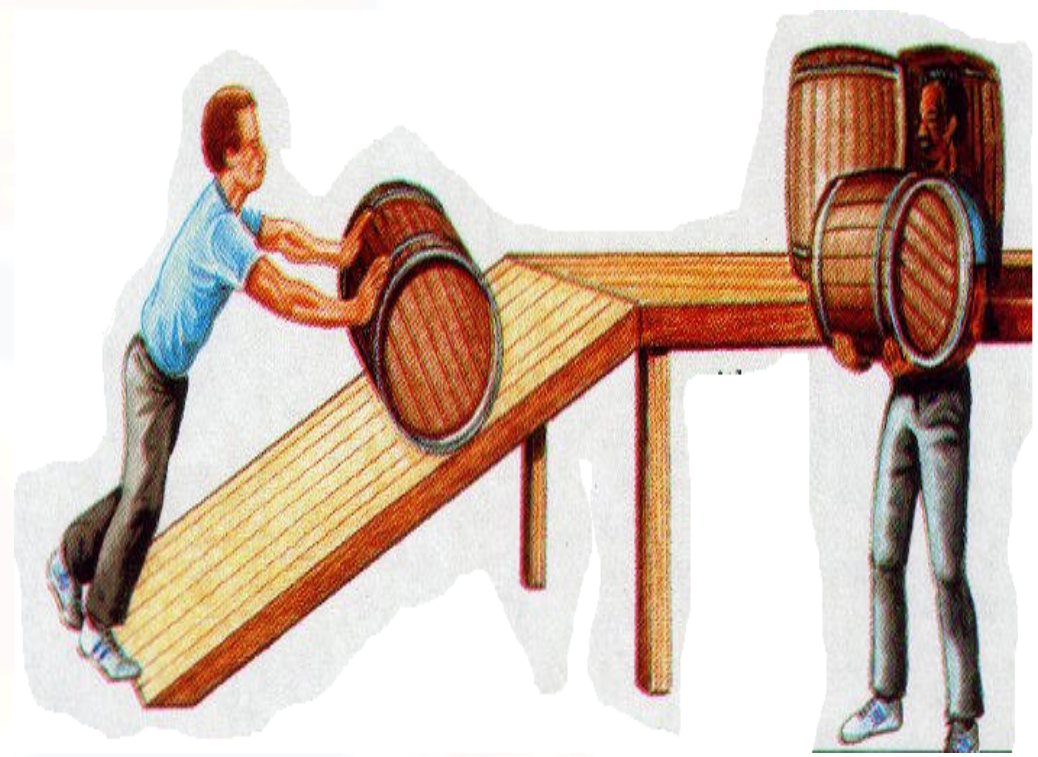
## c) Tension in the strings

$$
\begin{aligned}
T &= m_1 a \\
&= 3.80 \times 6.42 \\
&= 24.4 N
\end{aligned}
$$

# Inclined plane

**Inclined planes are used to help move heavy objects up a certain height**





# Strategies for solving problem related to inclined plane

1) Draw the free-body diagram.
2) Choose a coordinate system with $x$ parallel to incline plane.
3) Resolve $mg$ into components
4) Add vectors perpendicular to plane and set

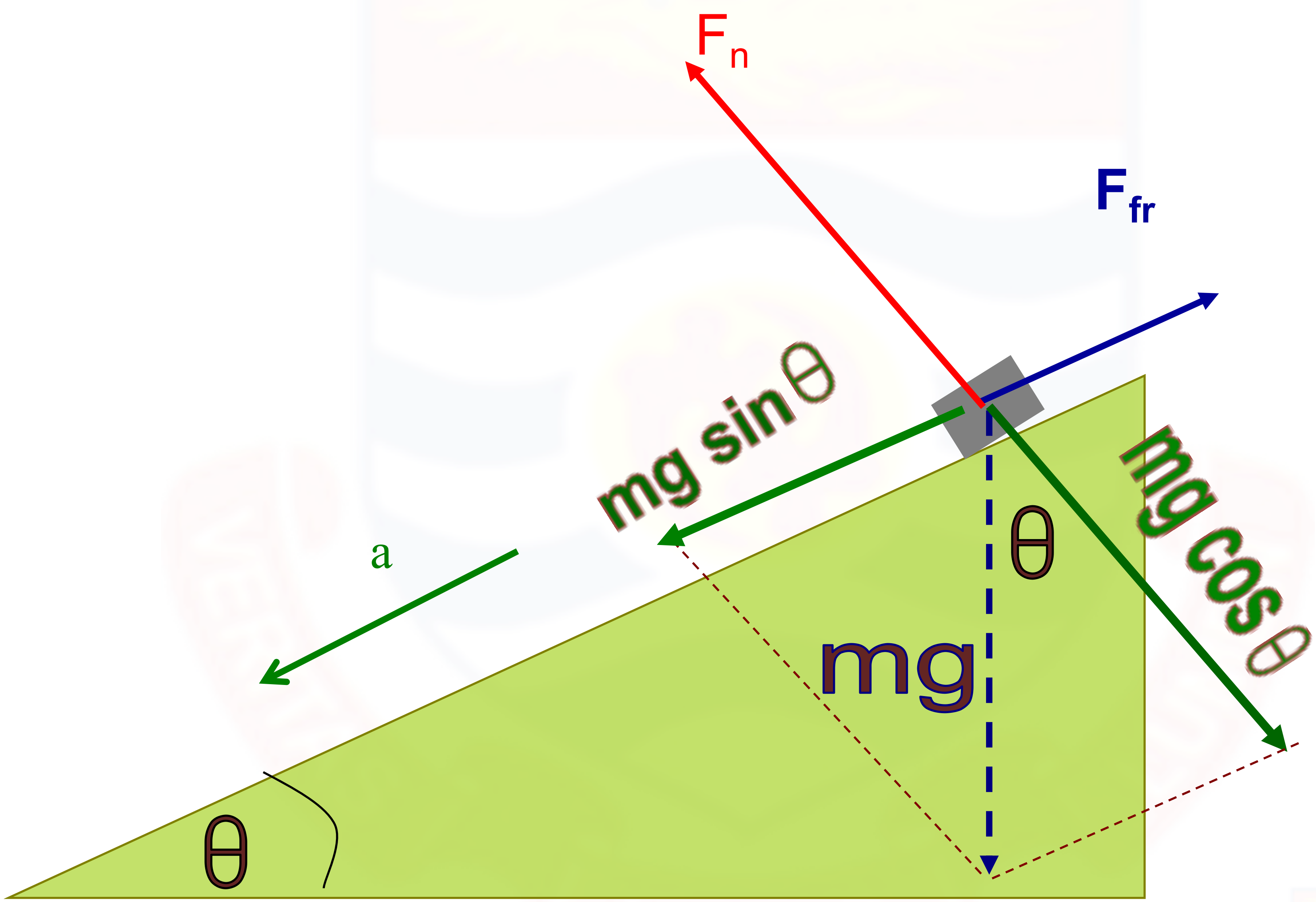
$$F_{net} = ma = 0$$

1) Add vectors parallel to the plane and set

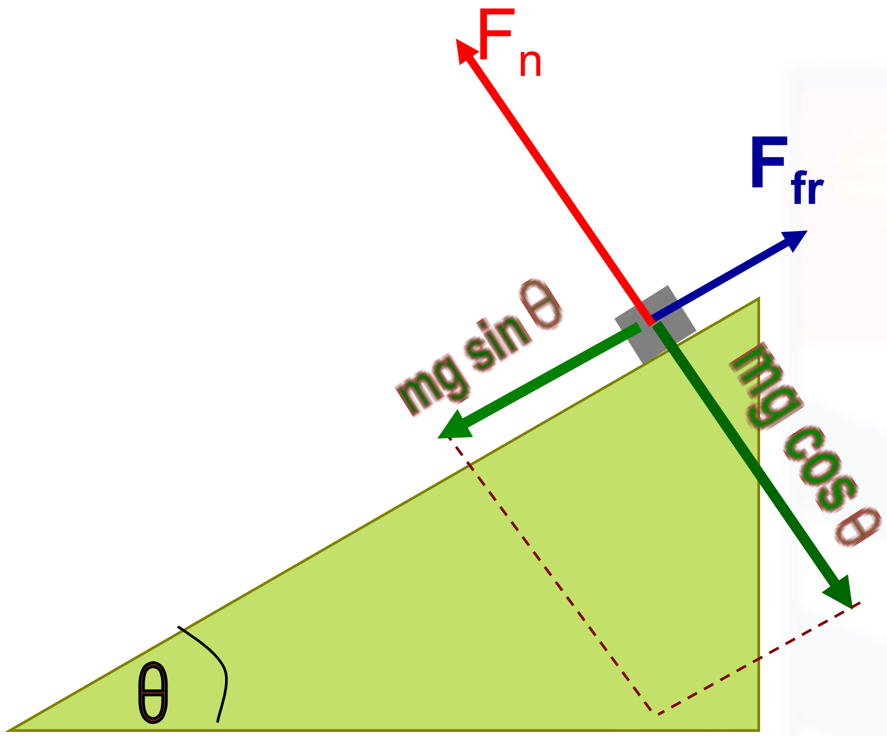
$$F_{net} = ma$$

# Forces on an Inclined Plane

The only force that we have to resolve into components is weight



# Forces on an Inclined Plane




Direction perpendicular to the incline plane:

$$F_{net} = ma = 0$$

$$F_n = mg\cos\theta$$

Force pressing the object into the surface is not full weight $mg$, but only part of it,

So, the normal force acting on the object is only part of full weight ($mg$)

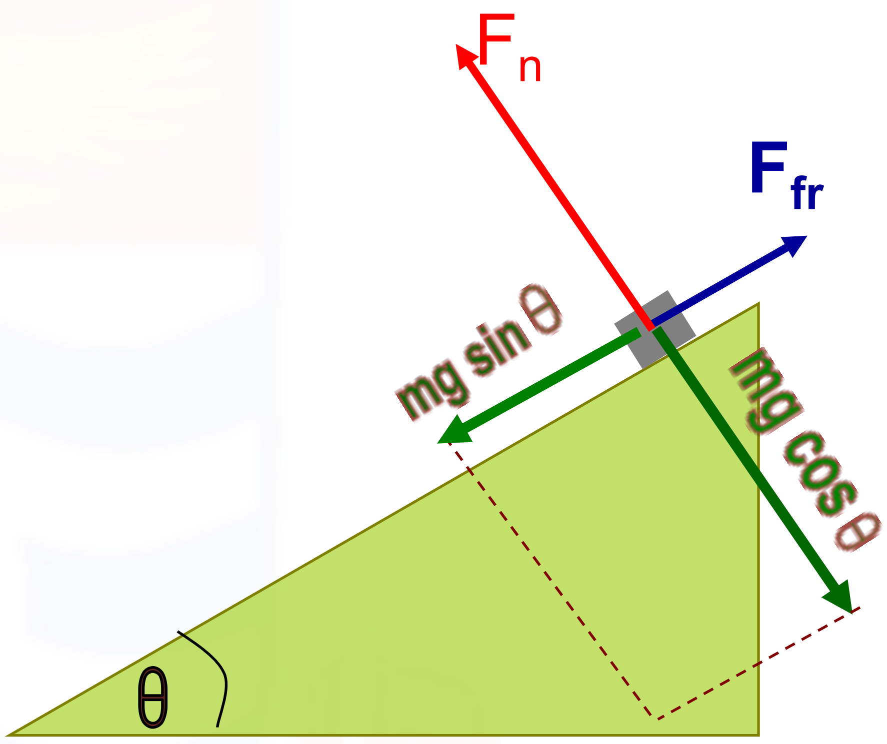
$$F_n = mg\cos\theta$$

# Forces on an Inclined Plane

- Direction parallel to the incline plane:

$$F_{net} = ma$$
$$F_f - mg\sin\theta = ma$$

- The force that causes acceleration downward is only part of the full force of gravity.
- The greater acceleration, the steeper the slope.
- If the incline = 0, then there is no horizontal movement due to gravity.




- $\mu_s = \dfrac{Frictional\ force}{Normal\ Force} = \dfrac{F_r}{F_N} = \dfrac{F_r}{mg\cos\theta}$
- $F_r = \mu_s F_N$

# Example

A particle, 5kg slides down the plane inclined at $30^0$. Calculate the:

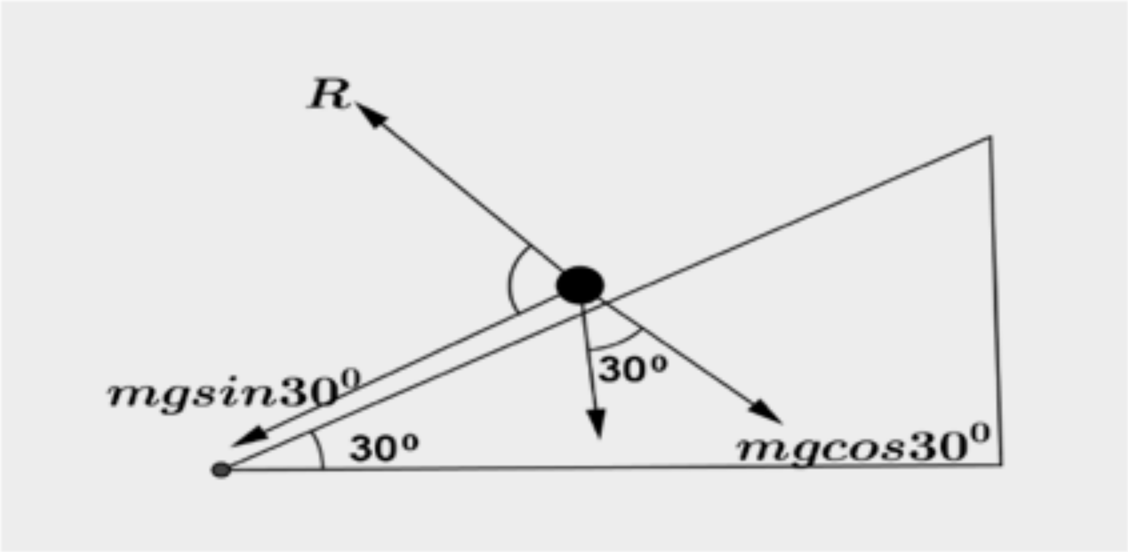
(a) normal reaction force

(b) force the particle slides down with

(c) acceleration of the particle

$[g = 10 \text{ m/s}^2]$

Mass of particle , m = 5 kg




The normal reaction force $F_N$, is given by

$$F_N = mgcos30^0$$
$$= 5 \times 10 \times 0.866 = 43.3 \text{ N}$$

The force F, down the plane is given by

$$F = mgsin30^0$$
$$5 \times 10 \times 0.5 = 25.0 \text{ N}$$

The acceleration, $a$ of the body is determined as

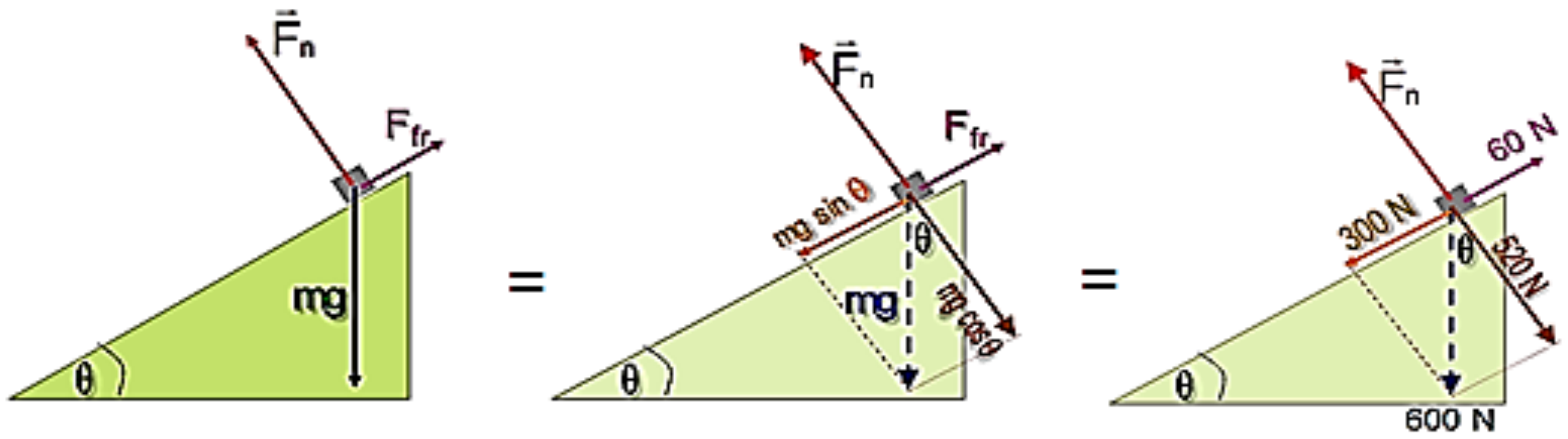
$$F = ma$$
$$25 = 5a = 5 \text{ ms}^{-2}$$

# Inclined Plane

A cute bear, m = 60 kg, is sliding down an iced incline plane at $30^0$. The ice can support up to 550 N. Will the bear fall through the ice? If the coefficient of the friction is 0.115, what is the acceleration of the bear?

# Inclined Plane

A cute bear, $m = 60\ kg$, is sliding down an iced incline $30^0$. The ice can support up to 550 N. Will the bear fall through the ice? If the coefficient of the friction is 0.115, what is the acceleration of the bear?




$m = 60$ kg
$\vartheta = 30^0$
$\mu = 0.115$
$g = 10$ m/s$^2$

$$F_{fr} = \mu F_n$$
$$= 0.115(520)$$
$$= 60\ \text{N}$$

**Perpendicular direction:**

$$F_{net} = ma$$
$$a = 0$$
$$F_n - mg\cos\theta = 0$$
$$F_n = 520\ \text{N} < 550\ \text{N}$$

The ice can support the bear.

**Parallel direction:**

$$F_{net} = ma$$
$$mg\sin\theta - F_{fr} = ma$$
$$300 - 60 = 60\,a$$
$$a = 4\ \text{m/s}^2$$

The cute bear is speeding up.

# Inclined Plane – You Do

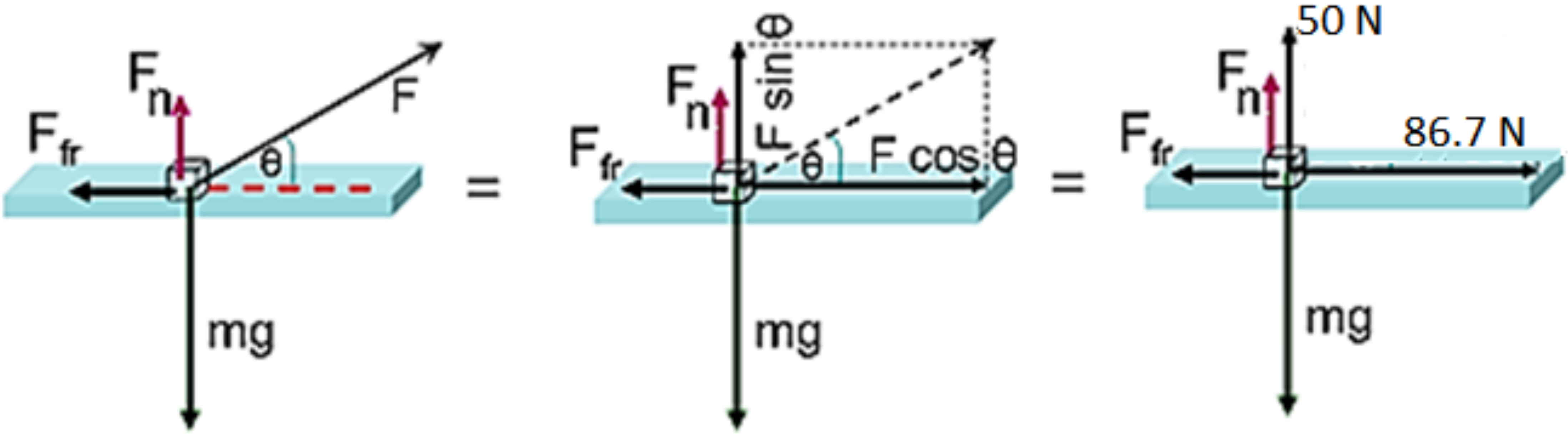
75 kg box slides down a ramp inclined at $25^{O}$ with an acceleration of 3.60 $m/s^2$.

a. Find the coefficient of friction. **(Ans: 0.069)**

b. What acceleration would a 175 kg box have on this ramp? **(Ans: 1.5 $m/s^2$)**

# Forces at an angle

Luke Skywalker starts to pull a sled with Princess Leia across a large ice pond with the force of 100 N at an angle of 30.0° with the horizontal. Find normal force and initial acceleration if the weight of sled and Princess Leia is 800 N and the friction force is 40 N.



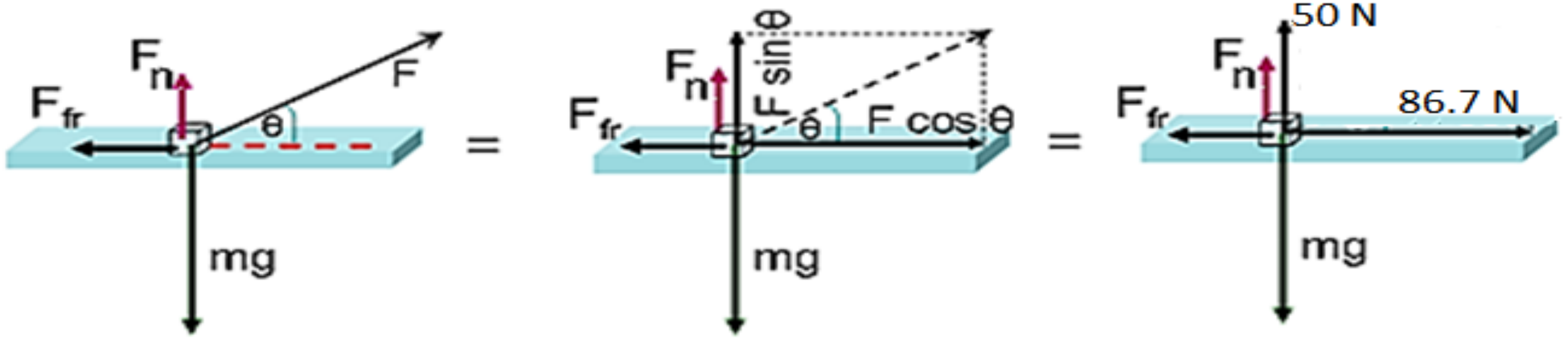

Add the forces in ***each direction***. What should the forces equal in

Horizontal forces: $F_{net} = ma$

Vertical forces: $F_{net} = 0$.

# Forces at an angle

Mensah starts to pull block across a large ice pond with the force of 100 N at an angle of 30.0° with the horizontal. Find normal force and initial acceleration if the weight of the block is 800 N and the friction force is 40 N.




$mg = 800$ N   $m = 80$ kg   $F = 100$ N   $F_{fr} = 40$ N

<u>Horizontal direction:</u>

$F \cos\theta - F_{fr} = ma$

$86.6 - 40 = 80\,a$

$a = 0.58$ m/s²

<u>vertical direction :</u>

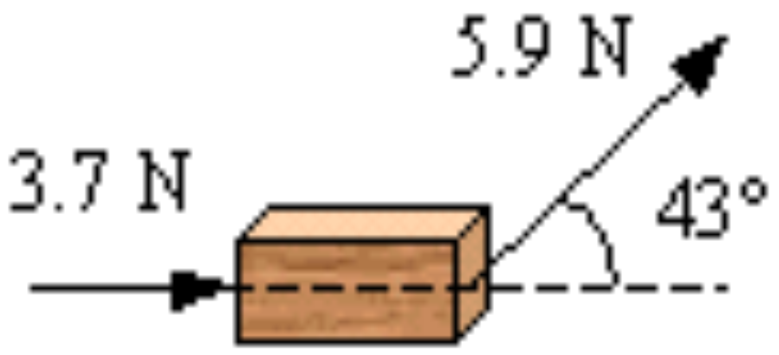
$F \sin\theta + F_n - mg = 0$

$50 + F_n = 800$

$F_n = 750$ N

# Forces at an angle

Two forces act on a 4.5-kg block resting on a frictionless surface as shown. What is the magnitude of the horizontal acceleration of the block?




Resolve into components, Add the forces, then find $F_{net} = ma$.

$3.7 + 5.9 \cos 43^0 = ma$

$8.0149 = 4.5a$

$a = 8.0149 / 4.5$

$= 1.7 \text{ m/s}^2$

# Motion under uniform acceleration

## (Equation of Motion-Kinematics)

$V = u + a\,t$

$\bar{v} = (v + u\ )\,/\,2$ (Average velocity)

$s = u\,t + ½\ a\,t^{\ 2}$

$v^2 = u^2 + 2\,a\,s$

$a = \bar{v}/t$

$\bar{v} = s/t$

$s = 1/2(v\ +\ u\ )t$

# Worked example

- A body starts with a speed 5 m/s and 5 s later its speed becomes 35 m/s. Find its acceleration and the distance travelled within this time interval.

$$u = 5\ \text{m/s} \quad v = 35\ \text{m/s} \qquad t = 5\ \text{s}$$

We are to determine $a = ?, s = ?$

Using the equation $v = u + at$

$$35 = 5 + 5a$$

$$a = 6\ \text{m/s}^2$$

Using the equation

$$s = 1/2(v + u)t$$

$$s = 1/2(35 + 5) = 100\ \text{m}$$

# Vertical Motion under gravity

When a particle is projected vertically upwards and if air resistance is neglected, then the acceleration of the particle is only gravity, $g$. Since $g$ offers resistance to the motion, we have

$$a = -g$$

$$v = u - g\,t$$

$$h = u\,t - \tfrac{1}{2}gt^2$$

$$v^2 = u^2 - 2\,g\,s$$

## worked example

A body is projected vertically upwards at 24 m/s.

(a) When will it attain maximum height?

(b) What maximum height will it attain?

(c) When will it pass its starting point and with what speed?

(d) When will it be 29.4 m below its starting point?

# Worked example

(a) $u = 24.5$ m/s, $g = 9.8$, $v = 0$ (At maximum height).

$$v = u - g\,t$$

$$0 = 24.5 - 9.8t$$

$$t = 2.5 \text{ seconds}$$

(b) $h = u\,t - \frac{1}{2}gt^2$

$$h = 24.5(2.5) - \tfrac{1}{2}(9.8)(2.5)^2$$

$$h = 30.625 \text{ m}$$

(c) The body passes its starting point when $h = 0$

$$24.5t - \tfrac{1}{2}(9.8)t^2 = 0$$

$$245t - 49t^2 = 0$$

$$t = 0 \text{ or } t = 5$$

So, the body passes its starting 5 second after projection.

# Worked example

(d) When h=-29.4, we have

$$24.5t - \tfrac{1}{2}(9.8)t^2 = -29.4$$

$$t = 6 \text{ or } t = -1$$

Since time cannot be negative, the body will be 29.4 m below it starting point 6 seconds after projection.

# Linear momentum

- **Linear momentum** is defined as the product of an object's mass and its velocity.

$$linear\ momentum\ (p) = mass \times velocity(v)$$

$$p = mv$$

- We usually just say momentum.

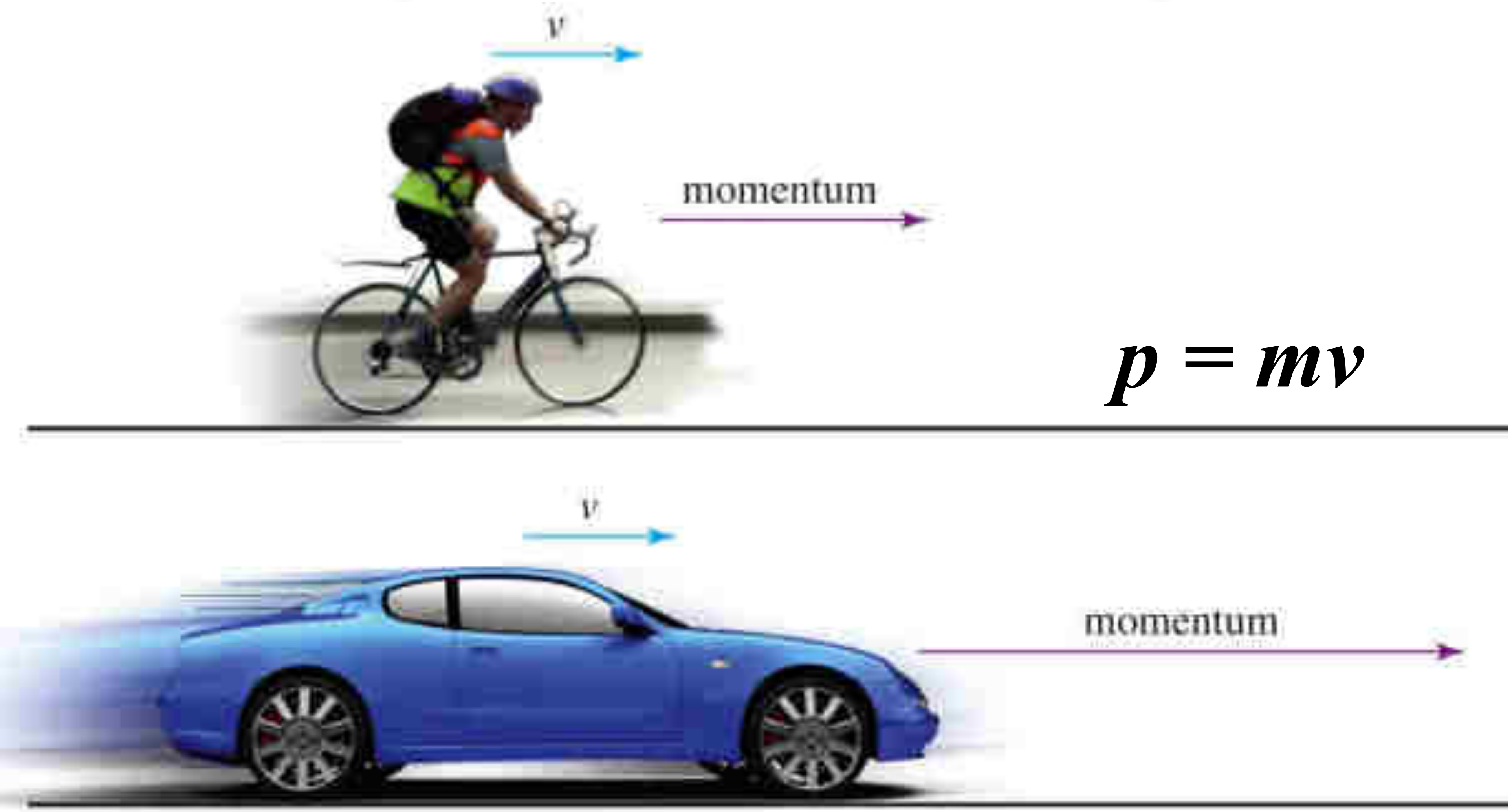
# Linear momentum

- Momentum is a measure of an object's state of motion.
- Consider an object whose momentum is 1 kg·m/s
  - This could be a 0.005 kg bullet traveling at 200 m/s.
  - This could be a 0.06 kg tennis ball traveling at 16.7 m/s.

$$p = mv$$

# Linear momentum

Car and bicycle have same speed but different momentum – because they have different masses.




$$p = mv$$

# Linear momentum

Newton's $2^{nd}$ law is closely related to momentum.

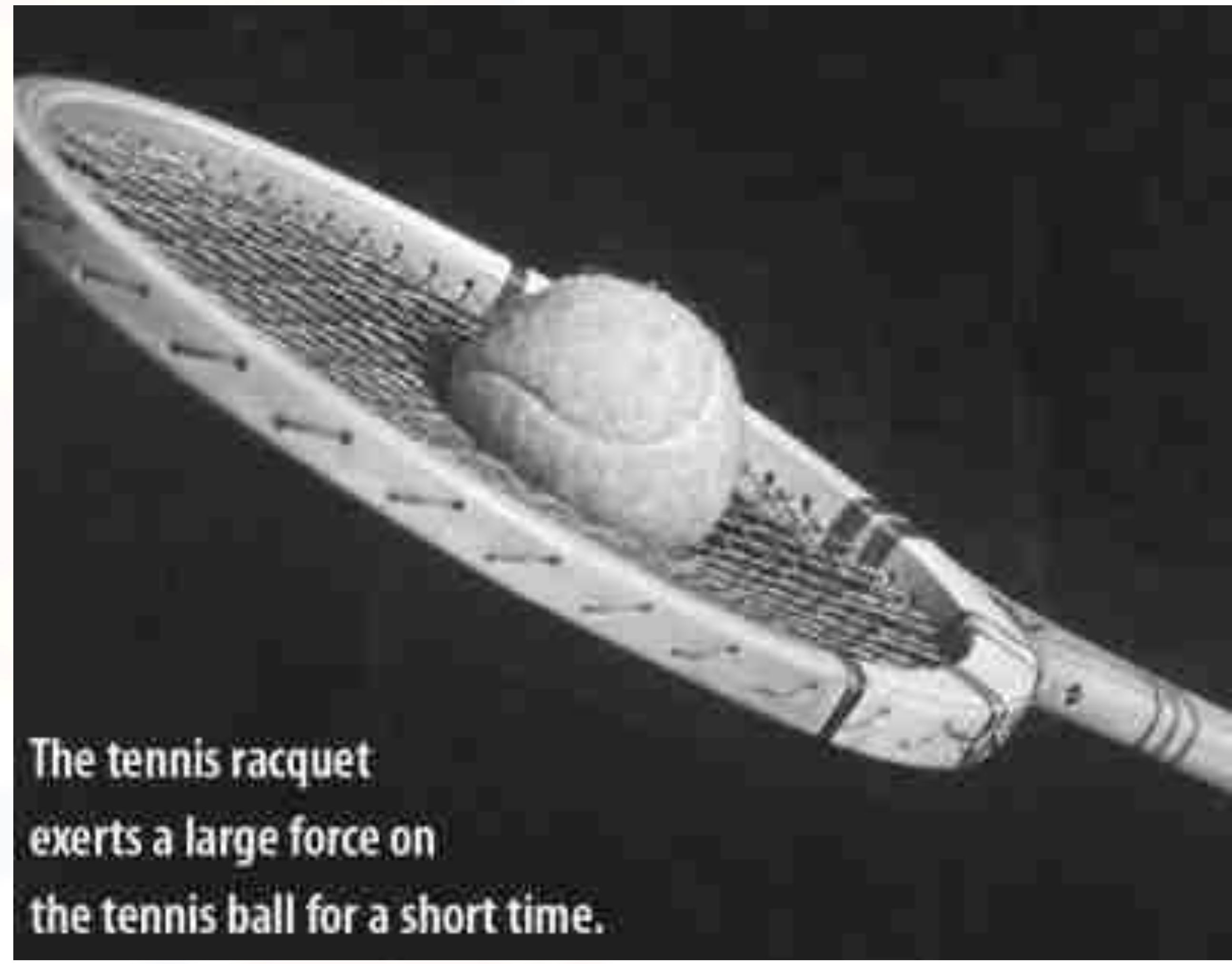
$$F = ma = m\frac{\Delta v}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = \frac{\Delta p}{\Delta t}$$

$$\text{force} = \frac{\text{change in momentum}}{\text{change in time}}$$

$$F = \frac{\Delta p}{\Delta t}$$

# Example

Estimate the average force on a tennis ball as it is served. The ball's mass is 0.06 kg and it leaves the racquet with a speed of 40 m/s. High-speed photography indicates that the contact time is about 5 milliseconds.


The tennis racquet exerts a large force on the tennis ball for a short time.

# Example

$$m = 0.06 \text{ kg}$$

$$v_i = 0 \text{ m/s}$$

$$v_f = 40 \text{ m/s}$$

$$t = 0.005 \text{ s}$$

$$F = \frac{\Delta(mv)}{\Delta t} = \frac{(0.06 \text{ kg})(40 \text{ m/s})}{0.005 \text{ s}}$$

$$= 480 \text{ N} = 108 \text{ lb}$$

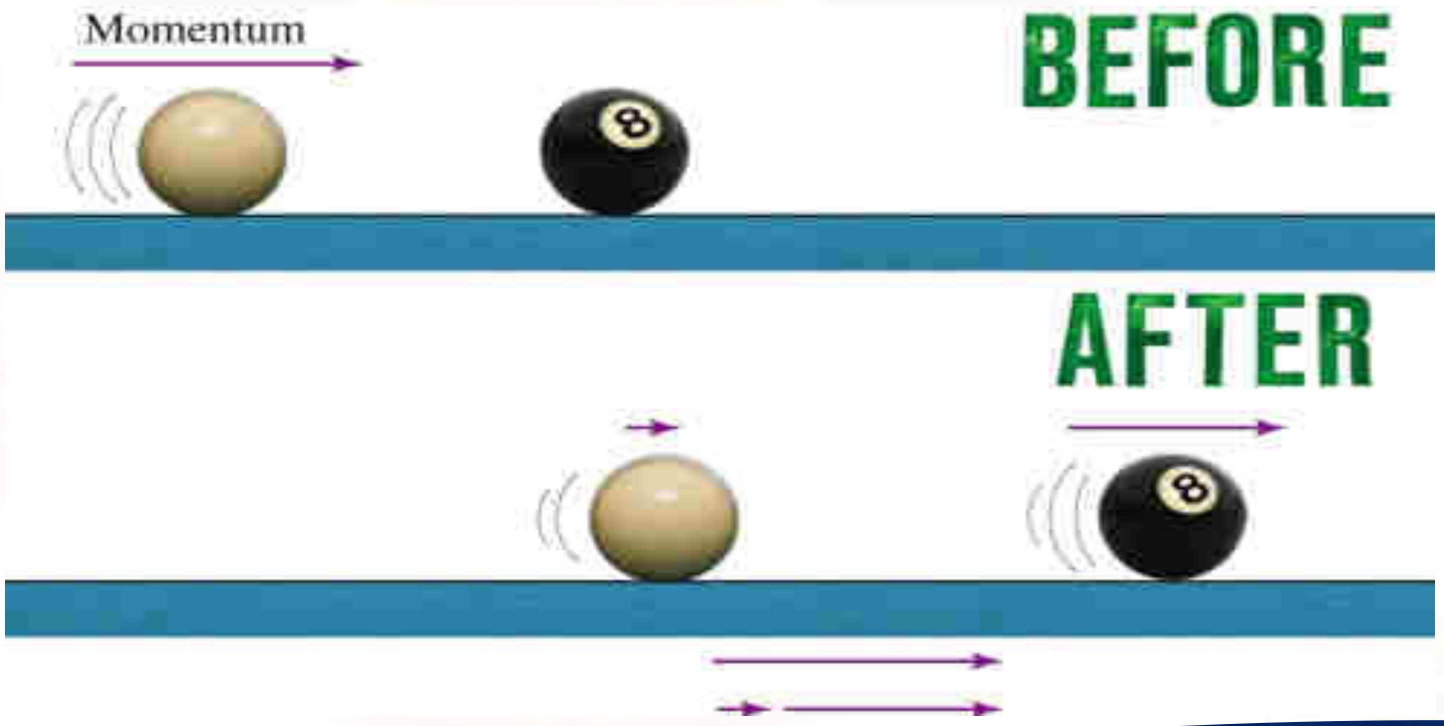
# Conservation of linear momentum

- The **Law of Conservation of Linear Momentum** states that the total linear momentum of an isolated system is constant.
- Isolated implies no external force:

$$F = 0 = \frac{\Delta p}{\Delta t} \quad \Rightarrow \quad \Delta p = 0$$

# Conservation of linear momentum

- This law helps us deal with collisions.
- If the system's momentum can not change, the momentum before the collision must equal that after the collision.



# Conservation of linear momentum

- We can write this as:

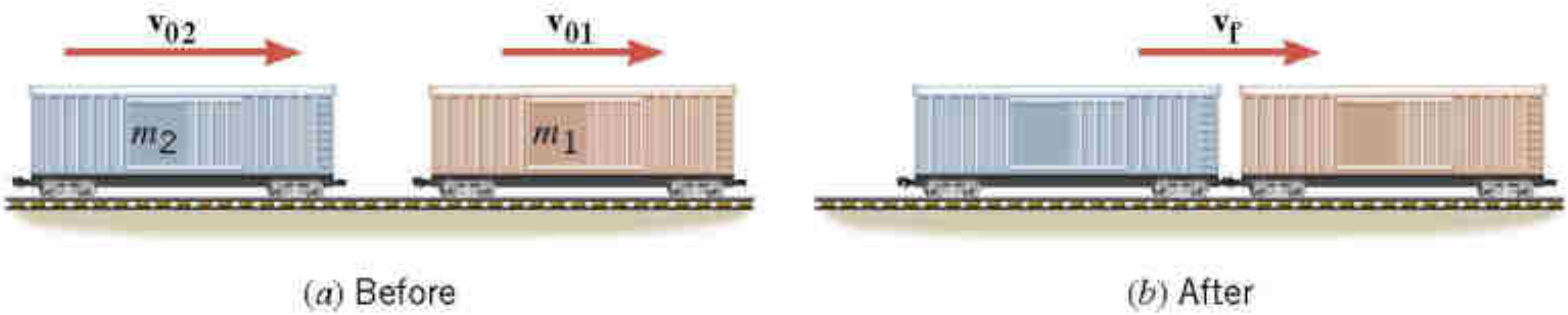
$$p_{before} = p_{after}$$

- To study a collision:
  - Add the momenta of the objects before the collision.
  - Add the momenta after the collision.
  - the two sums must be equal.

# Conservation of linear momentum

Two railroad cars with mass $m_1$ and $m_2$ collide, link together and move in tandem. The total momentum of the system remains the same.

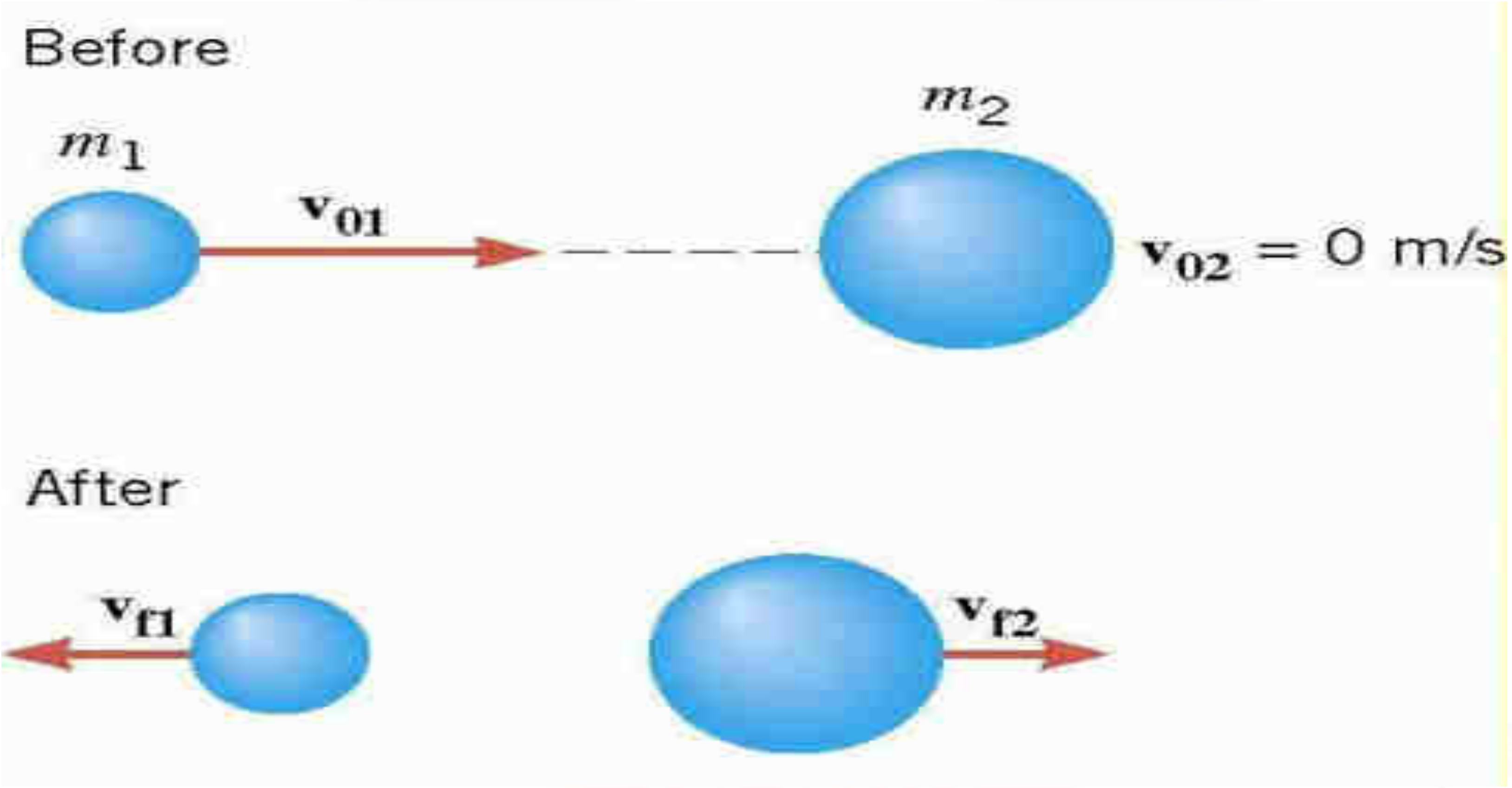
- $m_2v_{02} + m_1v_{01} = (m_2 + m_1)v_f$



# Conservation of linear momentum

Small ball with mass $m_1$ collides with stationary larger ball of mass $m_2$. The total momentum is the same before and after.

$m_1 v_{01} = m_2 v_{f2} - m_1 v_{f1}$

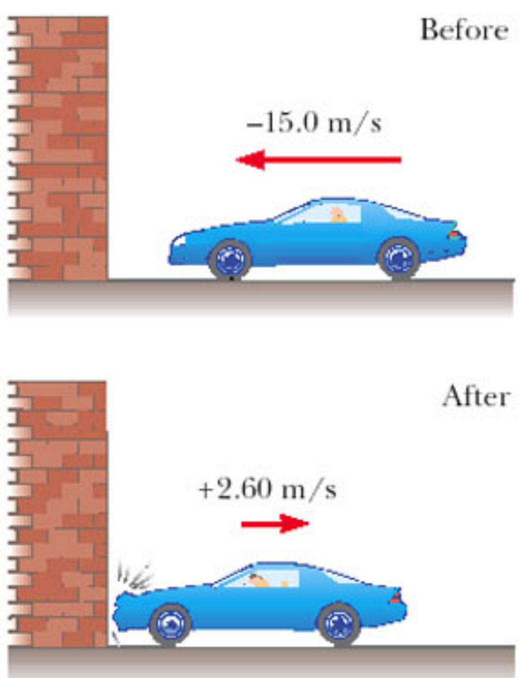


# How Good Are the Bumpers?

In a crash test, a car of mass $1.5\times10^3$ kg collides with a wall and rebounds as in the figure shown. The initial and final velocities of the car are $v_i$=-15 m/s and $v_f$ = 2.6 m/s, respectively. If the collision lasts for 0.15 s, find

(a) the impulse delivered to the car due to the collision

(b) the size and direction of the average force exerted on the car




# How Good Are the Bumpers?

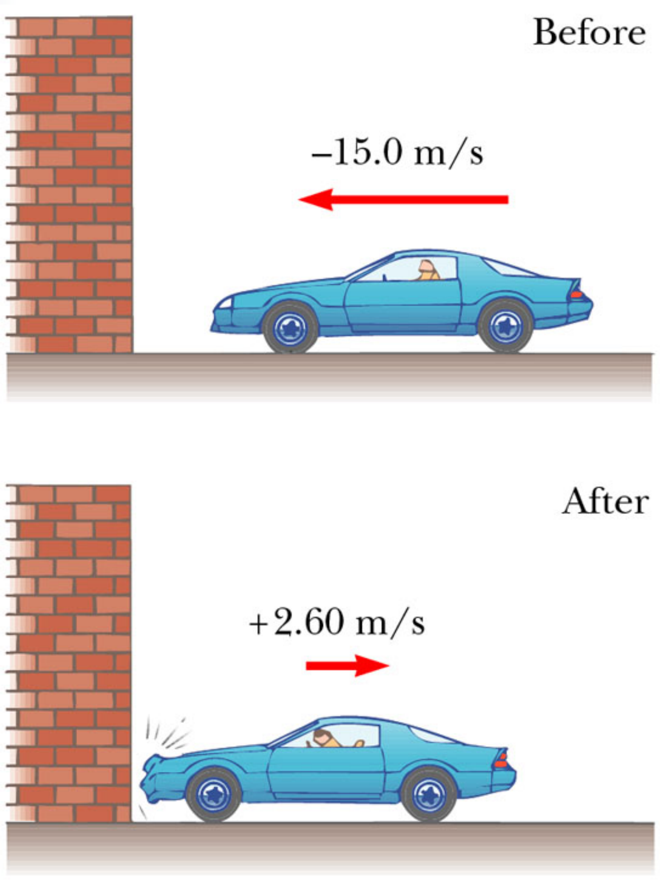
- In a crash test, a car of mass $1.5\times10^3$ kg collides with a wall and rebounds as in figure. The initial and final velocities of the car are $v_i$=-15 m/s and $v_f$ = 2.6 m/s, respectively. If the collision lasts for 0.15 s, find
(a) the impulse delivered to the car due to the collision
(b) the size and direction of the average force exerted on the car

$$p_i = mv_i = (1.5\times10^3 kg)(-15 m/s) = -2.25\times10^4 kg\cdot m/s$$

$$p_f = mv_f = (1.5\times10^3 kg)(+2.6 m/s) = +0.39\times10^4 kg\cdot m/s$$

$$I = p_f - p_i = mv_f - mv_i$$
$$= (0.39\times10^4 kg\cdot m/s) - (-2.25\times10^4 kg\cdot m/s)$$
$$= 2.64\times10^4 kg\cdot m/s$$

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{I}{\Delta t} = \frac{2.64\times10^4 kg\cdot m/s}{0.15 s} = 1.76\times10^5 N$$



# The Archer

An archer stands at rest on frictionless ice and fires a 0.5-kg arrow horizontally at 50.0 m/s. The combined mass of the archer and bow is 60.0 kg. With what velocity does the archer move across the ice after firing the arrow?

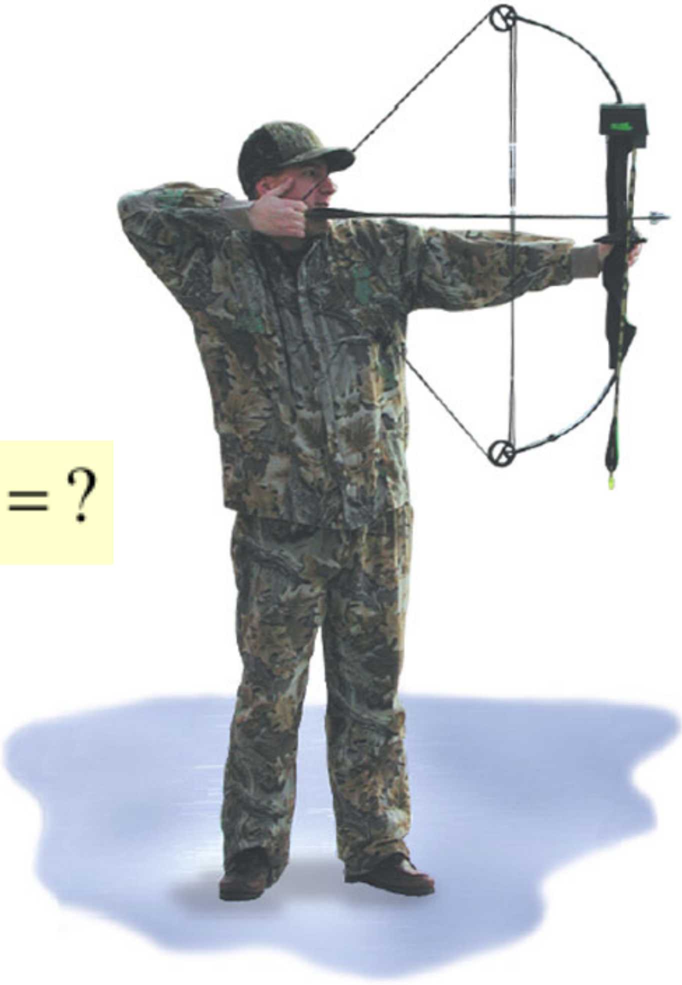
$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 = 60.0kg, m_2 = 0.5kg, v_{1i} = v_{2i} = 0, v_{2f} = 50m/s, v_{1f} = ?$$

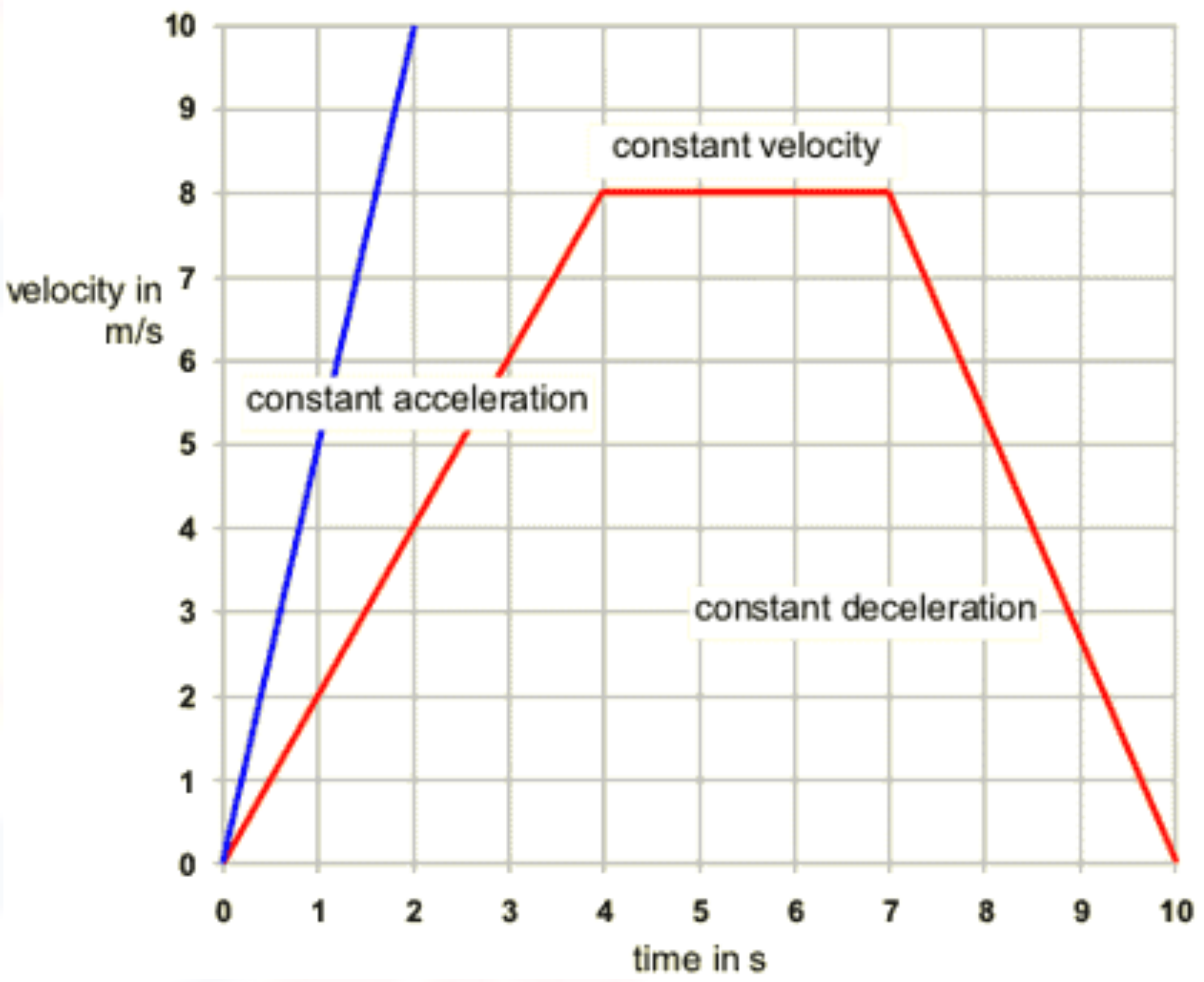
$$0 = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1f} = -\frac{m_2}{m_1} v_{2f} = -\frac{0.5kg}{60.0kg}(50.0m/s) = -0.417m/s$$

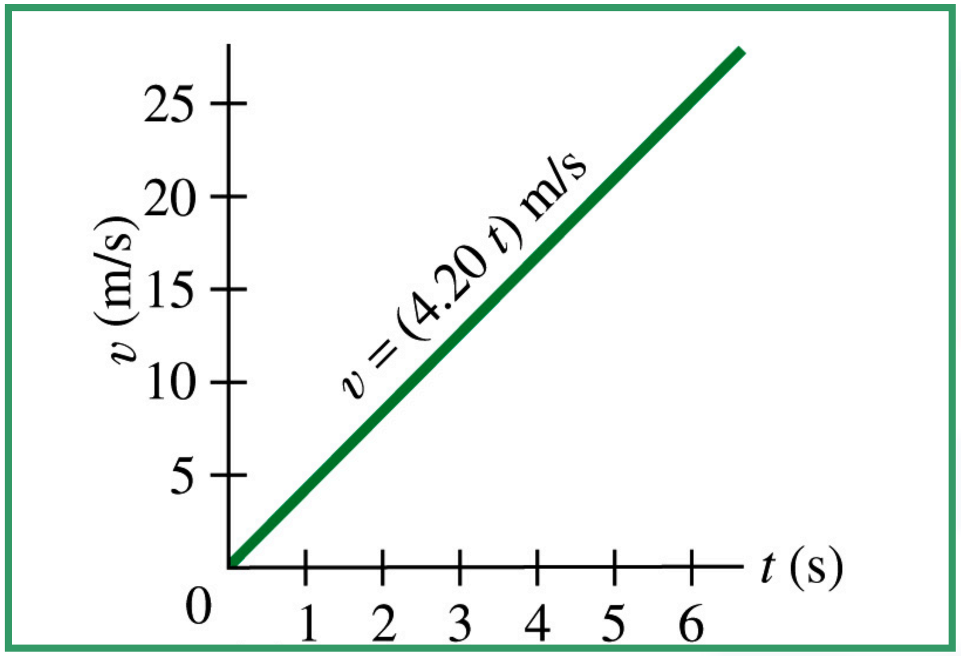


# Velocity- Time Graphs

- Velocity on y-axis
- Time on x-axis
- Displacement is the area under the curve.
- Slope is acceleration.
- Instantaneous Velocity is any point on the line.



# Velocity- Time Graphs

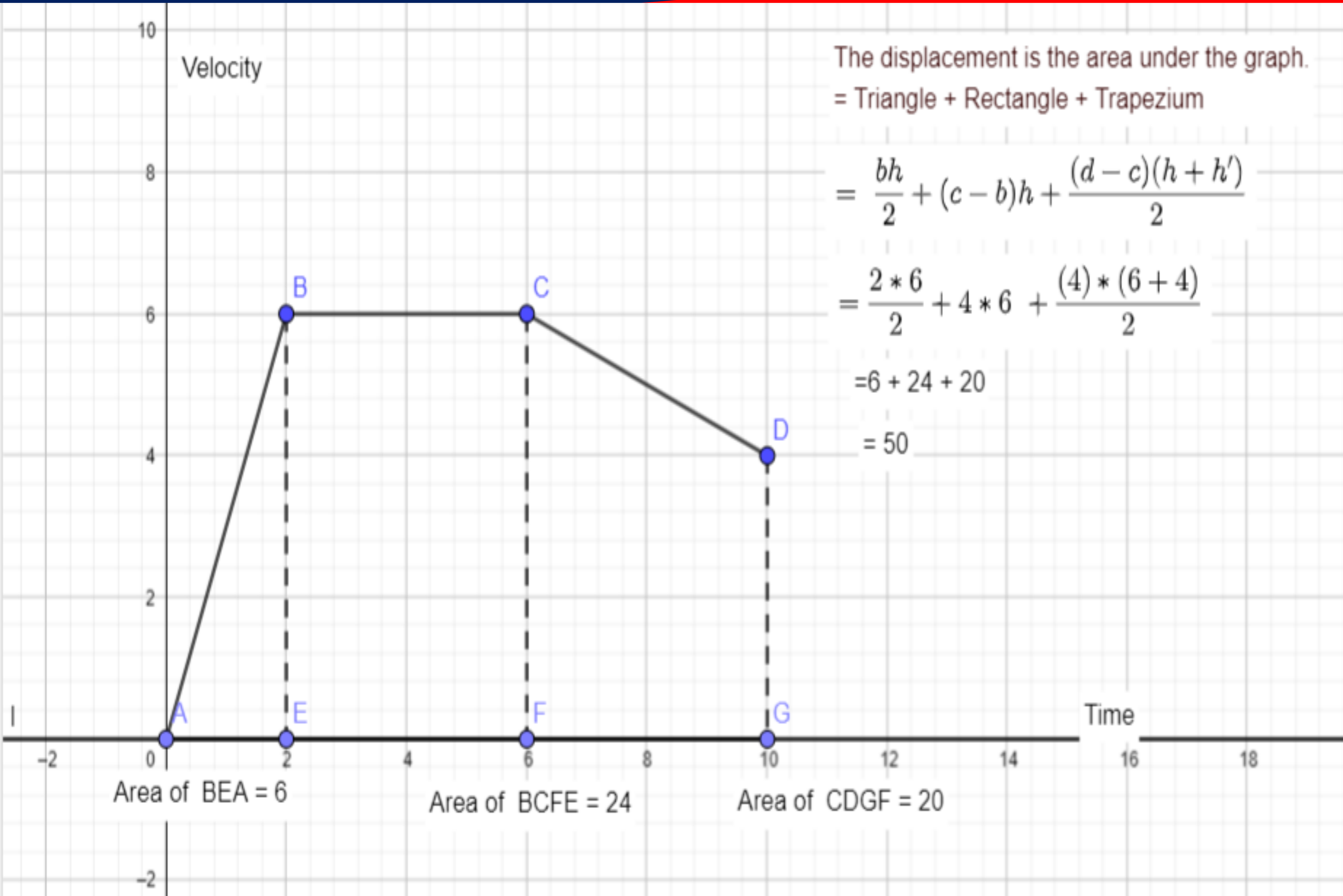



- Slope equals the acceleration

$$slope = 4.2m/s^2$$

- Area under the curve equals the displacement (distance)

$$area = \frac{1}{2}bh = \frac{1}{2}(6s)(25m/s) = 150m$$




The displacement is the area under the graph.

= Triangle + Rectangle + Trapezium

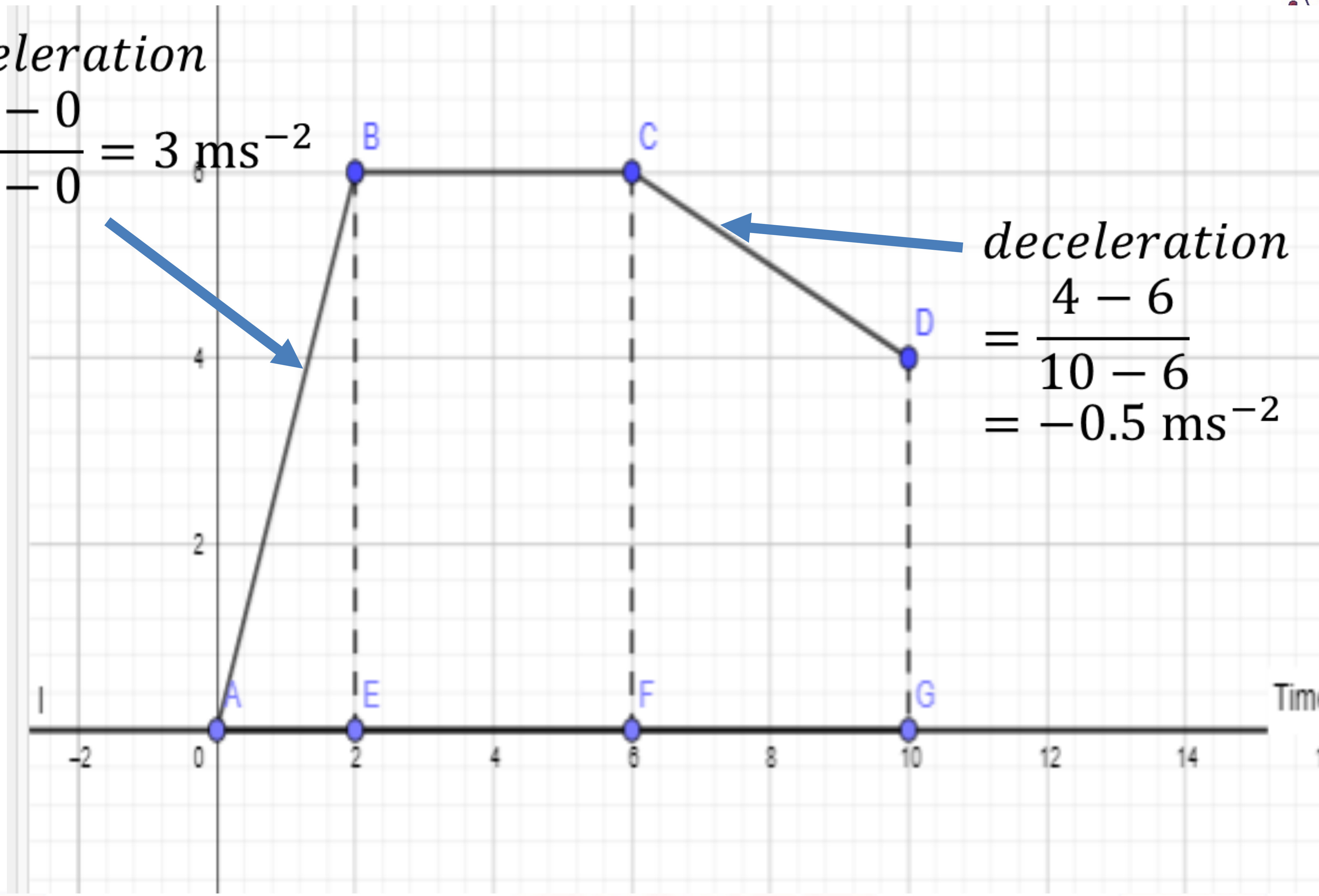
$$= \frac{bh}{2} + (c-b)h + \frac{(d-c)(h+h')}{2}$$

$$= \frac{2*6}{2} + 4*6 + \frac{(4)*(6+4)}{2}$$

=6 + 24 + 20

= 50

$$acceleration = \frac{6-0}{2-0} = 3\ \text{ms}^{-2}$$




$$deceleration = \frac{4-6}{10-6} = -0.5\ \text{ms}^{-2}$$