Module for
Certificate in Education Programme

EIN054SW:

INTRODUCTION TO ELEMENTARY MATHEMATICS

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UNIT 1: ARITHMETIC PROGRESSION

This unit introduces sequence known as arithmetic progressions (APs) and the corresponding series known as Arithmetic series. To master the techniques explained here it is vital that you undertake plenty of practice exercises so that they become second nature. After reading this text, and some video tutorial on this topic, you would be able to: recognise the difference between an arithmetic sequence and an arithmetic series; recognise an arithmetic progression; find the *n*-th term of an arithmetic progression; and finally find the sum of an arithmetic series.

Learning outcome(s)

By the end of the unit, you will be able to:

1. identify, define, and explain an Arithmetic Progression/Sequence and Series.
2. find the general term (*n* term) and the sum of the first n-terms of an Arithmetic Progression (A.P.).
3. apply knowledge gained to solve real life problems involving Arithmetic Progression (AP).

SESSION 1: IDENTIFYING, DEFINING, AND EXPLAINING AN ARITHMETIC PROGRESSION AND SERIES

In this session we will focus on examples of arithmetic progression, identify progressions which are arithmetic progressions and those which are not and finally, define an arithmetic progression. We shall also define an arithmetic series.

Learning outcomes

By the end of the session, you will be able to:

1. identify and define an arithmetic progression and series
2. find the nth term and sum of the first n terms of an arithmetic progression.
3. Solve real life problems involving the use of arithmetic progression and series.

1.1 Identifying and defining an arithmetic progression and series

What is an Arithmetic Progression (AP)? It is a set of numbers which are written in some order such that there is a constant difference between the successive terms of the sequence. For example, take the numbers 1, 3, 5, 7, 9, ... Here, we seem to have a rule. We have a sequence of odd numbers. To put this another way, we start with the number 1, which is an odd number, and then each successive term is obtained by adding 2 to give the next odd number. Similarly, the sequence 9, 6, 3, 0, -3, ….. is an arithmetic progression because the difference between successive terms is the constant, -3. Here is another sequence: 1, 4, 9, 16, 25, ... This sequence is **not** an arithmetic sequence or progression, why? And this sequence, 1, −1, 1, −1, 1, −1, . . . , is also **not** an arithmetic sequence or progression because the successive terms of numbers alternate between 1 and −1. Think about two arithmetic sequences of your own and write them down.

An arithmetic series is something we obtain from a sequence by adding all the terms of the sequence together. For example, the series from the arithmetic progression: 1, 3, 5, 7, 9, … is 1+3+5+7+9+…. The series we obtain from the sequence 9, 6, 3, 0, -3 …… is 9+6+0±3+… . Note that a sequence which does not end (shown by the dots) and we write Sn for the sum of these n terms. So, although the ideas of a **sequence** and a **series** are related, there is an important distinction between them. For example, let us consider the sequence of numbers 1, 2, 3, 4, 5, 6, . . ., *n*. Then S1 = 1, as it is the sum of just the first term on its own. The sum of the first two terms is $S_2 = 1 + 2 = 3$. Continuing, we get $S_3 = 1 + 2 + 3 = 6$, $S_4 = 1 + 2 + 3 + 4 = 10$, and so on.

Arithmetic progression is defined as the sequence of numbers in algebra such that the difference between every successive term is the same. It can be obtained by adding a fixed number to each previous term.

1.2 Finding the *n*th term and sum of the first *n* terms of an arithmetic progression.

a. Finding the *n*th term of an Arithmetic Progression (AP)

In an arithmetic progression, there is a possibility to derive a formula for the n^{th} term of the AP. For example, the sequence 2, 6, 10, 14, … is an arithmetic progression (AP) because it follows a pattern where each number is obtained by adding 4 to the previous term. In this sequence, n^{th} term = 4n-2. The terms of the sequence can be obtained by substituting n=1, 2, 3, ... in the n^{th} term. i.e.,

- When n = 1, $4n-2 = 4(1)-2 = 4-2=2$
- When n = 2, $4n-2 = 4(2)-2 = 8-2=6$
- When n = 3, $4n-2 = 4(3)-2 = 12-2=10$

The first number in a sequence is called **first term. You should realise that** a sequence may be **finite** or **infinite**. A finite linear sequence terminates, and the last number is called **last term**.

In general, the sequence a, $a+d$, $a+2d$, $a+3d$, …, l is an example of a linear sequence or Arithmetic progression with first term a, common difference d and last term l.

By a careful study of the Table 1, you will discover that the nth term of any linear sequence is given by a+n-1d, where n=1, 2, 3 ....

Term	Rule
1st	a
2nd	a+d
3rd	a+2d
4th	a+3d
***n*th**	a+(n-1)d

Table 1

We can from the above conclude that the last term of an AP, l with n terms is also given by:
$l=a+(n-1)d$

b. Finding the sum of the first *n* terms (S_n) of an arithmetic progression (Arithmetic Series)

Considering the finite arithmetic series 1+3+5+7+9+11+13+15, we can find the sum quickly with less difficulty by going through the following steps:

Step 1: Let $S=1+3+5+7+9+11+13+15$ (reverse the series)
Step 2: Reverse the series; $S=15+13+11+9+7+5+3+1$
Step 3: Find the sum of the two expressions; $2S=16+16+16+16+16+16+16+16$
Step 4: Use the idea of multiplication as repeated addition; $2S=8\times 16$
Step 5: Solve for *S*; $S=8\times 162=64$
Try this for other linear series.

Let us now generalise or attempt to find the rule for an *n*-term arithmetic series going through the steps above for the arithmetic series: $a+a+d+a+2d+\ldots+l$)
Step 1: Let $S=a+a+d+a+2d+\ldots+l$
Step 2: $S=l+\ldots+a+3d+a+2d+(a+d)\ldots+a$
Step 3: $2S=a+l+a+l+a+l+\ldots+(a+l)$
Step 4: $2S=na+l$
Step 5: $S=na+l2$ …………………………… equation (1)
Substituting $l=a+(n-1)d$ in $S=na+l2$, we obtain:
$S=n[a+a+(n-1d)]2$.

$S=n[2a+(n-1d]2$………………………… equation (2)

Let us now discover how equations (1) and (2) work using the arithmetic series:
$1+3+5+7+9+11+13+15$.

From the series, the first term, $a=1$ the last term, $l=15$, number of terms, $n=8$, and common difference $d=2$.

Thus, from equation (1), $S=81+152=8\times 162=64$
Also, from equation (2), $S=8[2\times 1+(8-12]2=8(2+14)2=8\times 162=64$
What do you notice? They worked. Well done

Generate your own arithmetic series and fine their sums.

1.3 Solving real-life problems involving the use of arithmetic progression and series.

We will introduce you to solving of real-life problems involving arithmetic progression and how you can employ the knowledge gained to solve. We shall do this using examples.

Example 1

A trader started with initial capital of GH¢4,000 in a business. The trader makes an average profit of GH¢1,500, monthly. How much will the trader be having after the first year in business?

Solution.

From the question the following data can be gathered:
First term, $a=$GH¢4,000, common difference, $d=$GH¢1,500, and the number of terms, $n=12$.

We are required to find how much the trader will be having after 1 year (12 months).
That is $l=a+11d$, where d, the common difference.

Thus, l=4,000+11×1,500=20,500
Thus, the trader will be having GH¢20,500 at the end of the year.

Example 2
A student makes a savings of GH¢115 every week. Assuming there are no deductions from his saving account, how much will be in in the student's savings account at the end of the year if there is an initial amount of GH¢235 in the account?

Solution

Data
Initial amount (first terms) a=GH¢235
Monthly savings (constant difference) d=GH¢115
Number of months (number of terms) n=52 weeks

Amount in account at the end of the year S= GH¢235+(52-1) ×52
Thus, S= GH¢235+51×52
= GH¢2887

Thus, there will be GH¢2887 in the student's account at the end of the year.

Key ideas/core points

- An Arithmetic sequence a set of numbers which are written in some order such that there is a constant difference between the successive terms of the sequence.
- An arithmetic sequence is of the form a, a+d, a+2d, a+3d, …a+n-1d.
  Where *a* is the first term, *d* is the common difference and *n* is the number of terms.
- The expression a+n-1d is the last term of the sequence (*l*) or the *n*th term of the sequence.
- An Arithmetic series is the sum of an arithmetic sequence a+a+d+a+2d+a+3d…+[a+n-1d]
- The sum of the first n-terms of an Arithmetic series is given by Sn=n[2a+(n-1d]2

UNIT 2: GEOMETRIC PROGRESSION

We shall now move on to another type of sequence we want to explore: Consider the sequence 2, 6, 18, 54,… Here, each term in the sequence is 3 times the previous term. And in the sequence 1, −2, 4, −8,… each term is −2 times the previous term.
Sequences such as these are called geometric progressions, or GPs for short.
By the end of this unit, you will be able to:

1. give examples and non-examples geometric progressions
2. state the general term and sum of a geometric progression
3. apply the general term and the sum of a geometric progression to solve real life problems

SESSION 1 GIVING EXAMPLES AND NON-EXAMPLES GEOMETRIC PROGRESSIONS

Learning Outcome(S)

By the end of the unit, you will be able to:

1. identify, define, and explain an exponential or a geometric progression (G.P.) and series;.

2. find the general term (*n* term) and the sum of the first n-terms of an exponential or a geometric Progression (G.P.);

3. apply knowledge gained to solve real life problems involving an exponential or a geometric progression (G.P.);

2.1 Identifying and defining an exponential or a geometric progression (G.P.)

What is an exponential or a geometric progression (G.P.)? It is a set of numbers which are written in some order such that there is a constant ratio between the successive terms of the sequence. For example, take the numbers 2, 4, 8, 16, ... Here, we seem to have a rule. We have a sequence of even numbers. To put this another way, we start with the number 2 and then each successive term is obtained by multiplying 2 to give the next even number. Similarly, the sequence 9, 3, 1, 13, 19, ….. is a geometric progression because the common ratio between successive terms is the constant, 13. Here is another sequence: 1, 4, 9, 16, 25, … This sequence is **not** a geometric progression, why? And this sequence, 1, −1, 1, −1, 1, −1, …, is also **not** a geometric progression because the successive terms of numbers alternate between 1 and −1. Think about two geometric progressions of your own and write them down.

A geometric series is something we obtain from a geometric sequence by adding all the terms of the sequence together. For example, the series from the geometric progression: 2, 4, 8, 16, 32, … is 2+4+8+16+32+…. The series we obtain from the sequence 9, 3, 1, 13, 19, ….. is
9+ 3+ 1+ 13+ 19+….. We write S_n for the sum of these n terms.

SESSION 2 STATING THE GENERAL TERM AND SUM OF A GEOMETRIC PROGRESSION

Learning outcome(s)

1. Find the *n*th term of an exponential or a geometric progression
2. Find the sum of the first *n* terms of an exponential or a geometric
3. Find the sum to infinity of an exponential or a geometric

2.1 Finding the *n*th term an exponential or a geometric progression.

a. Finding the *n*th term of a geometric progression

In a geometric progression, there is a possibility to derive a formula for the *n*th term of the progression. For example, the sequence 2, 4, 8, 16, … is a geometric progression (GP) because it follows a pattern where each number is obtained by multiplying 2 by to preceding terms to obtain consecutive terms or finding the ratio of successive terms. You will discover that the ratio of successive terms in a geometric sequence is a constant. This constant ratio is called common ratio. For instance, in the geometric progression 2, 4, 8, 16, … , the constant ratio is 2 since 21=42=84=168=2

2.2 Finding the sum of the first *n* terms of an exponential or a geometric progression.

Generalising, a geometric progression with first term, *a* and common ratio, *r* will have the sequence as , ar, ar2, ar3, … . Following the pattern in the terms of the sequence, one can conclude that the *n*th term of a geometric sequence Un=arn-1

b. We shall next attempt to find the sum of the first n terms of a geometric series.

For example, considering the geometric series 1+2+4+8+16+32+64+128,

Let S=1+2+4+8+16+32+64+128. ………………(1)

Multiplying through

Multiply S by 2, the common ratio, we obtain

2S=2+4+8+16+32+64+128+256……………….(2)

(1)-(2)

S-2S= 1-256

Simplifying

S=255

Therefore S=255

Generalising:

- Let Sn= a+ar+ar2+ar3+…+arn-1 ………(1)

Multiply Sn by r, the common ratio, we obtain

- rSn= ar+ar2+ar3+ar4+…+arn ……….(2)

1. - (2) gives

- Sn-rSn=a-arn

  Thus, Sn1-r=a(1-rn)

  Finally, Sn=a(1-rn)1-r, for r ≠ 1

  Let us work some examples.

Example 1

Find the sum of the geometric series
2 + 6 + 18 + 54 + . . .
Where, there are 6 terms in the series.

Solution
For this series, we have a = 2, r = 3 and n = 6. So
Substituting in Sn=a(1-rn)1-r, for r ≠ 1, we have:
S6=2(1-36)1-3, for r ≠ 1
ie. S6=2(-728)-2,
Hence, S6=728

Example 2
Find the sum of the geometric series
8 − 4 + 2 − 1 + . . .
Where, there are 5 terms in the series.

Solution

For this series, we have a = 8, r =-12 and n = 5.
Thus, S5=81--1251--12

Simplifying, S5=81+13232

Further, simplification S5=2×8×33323

Finally, S5=5.5

Example 3
How many terms are there in the geometric progression 2, 4, 8, . . . , 128 ?

Solution
In this sequence a = 2 and r = 2. We also know that the n-th term is 128. But the formula for the n-th term is arn-1. So
128=2×2n-1
64=2n-1
n=7
So there are 7 terms in the geometric progression.

With the help of a calculator, find the *n*th powers of 12, 1 4, and 15, where *n*= 2, 3, 4, 5, ..., 10.
What do you notice about the values of ½, 1/3, 1/5 as *n* tends larger?
I believe your response is that it becomes zero (0)

2.3 Finding the sum to infinity of an exponential or a geometric

Thus, from the rule Sn=a(1-rn)1-r, for r: $-1<r<1$, as n tends to infinity, rn becomes zero and therefore, Sum to infinity (S)

S=a1-r

Example 1
Find the sum to infinity of the geometric progression:

1, 13, 19, 127, …

Solution
First term, a=1 , common ratio, r=13
Therefore sum to infinity, S=11-13
S=123
S=32

SESSION 3. APPLYING THE GENERAL TERM AND THE SUM OF A GEOMETRIC PROGRESSION TO SOLVE REAL LIFE PROBLEMS

By the end of the unit, you will be able to:
1. Solve real-life problems involving the use of geometric progression and series.

3.1 Solving real-life problems involving the use of geometric progression and series.

The 4 situations listed below are real life situations which may require the use of geometric progression and series to solve them.

1. Dropping a ball from a height and the ball's height reducing by a third at every drop. Finding the height at the 6^{th} drop.

2. Calculating the amount of radioactive material left after any given number of half-lives of the material. During each half-life, the material decays by 50 percent.

3. Suppose you own a car that you purchased for some amount. If the price of the car depreciates (decreases) by same percentage every year, you can find out the value of the car after n years, using the geometric sequence.

4. If you put your money in a bank and the bank provides you a fix annual rate of interest, then you can calculate the amount you will have in your account after certain years by using the concept of geometric progression.

Key ideas/Core points
- The *n*th term of a geometric sequence Un=arn-1
- The sum of the first *n* terms of a GP, Sn=a(1-rn)1-r,
- Sum to infinity of a GP, S=a1-r

UNIT 3: STATISTICS I

In all the branches of mathematics, statistics seem to be the most visible one in the everyday world. Statistics is a collection of methods for planning experiments, gathering data, and then organising, analysing, interpreting, and representing data with the intent of drawing conclusions and making conclusions based on the data.

Data is a fact or figure that provides some information, usually gathered to support a position or for some sort of decision-making purpose. Statistical data is usually numerical information that can be presented in variety of forms including, whole numbers, fractions, decimal numbers, and percentages. Raw data are collected data that have not been organised numerically. Statistical data are obtained either by counting or measuring.

Learning outcomes

By the end of this unit, you will be able to:

1. Collect Data
2. Organize Data
3. Present Data

SESSION 1: COLLECTION OF DATA

Learning outcomes

By the end of this session, you will be able to:

1. Collect data by counting
2. Collect data by measuring

1. Collecting data by counting

Statistical data that can be collected by counting includes the following:

- Birthdays
- Birth months
- Birth years
- Marks obtained in a class test marked out of a certain total
- Favourite foods or drinks or games or hobbies or colours
- Throwing of a die 50 times

Just name a few. Data obtained are recorded

1.2 Collecting data by measuring

Statistical data that can be collected by measuring includes the following:

- Heights
- Masses
- Time taken to accomplish a task
- Temperatures
- Capacities

Data obtain through measurements are collected and recorded.

In both data collected by counting and measuring, the recorded data which are yet to be organised are called raw data.

Below are two examples of raw data collected by counting and measuring respectively.

1. Marks obtained by students in a mathematics test marked out of 20.

18	15	17	19	16	16	15
17	16	16	15	18	15	17
16	18	16	17	18	19	16
18	17	15	16	17	16	16
16	15	17	17	15	17	17

2. The data below are heights (in cm) of students in a school.

152	153	152	154	153	154
155	153	154	152	154	153
154	155	156	154	152	157
154	156	155	153	155	154
157	156	153	155	156	157

Data can be collected in many ways including the use of instruments such as the following:

- Questionnaire
- Interview guide
- Survey guide

Key ideas/core points

- Statistics is a collection of methods for planning experiments, gathering data, and then organising, analysing, interpreting, and representing data with the intent of drawing conclusions and making conclusions based on the data.
- Data is a fact or figure that provides some information, usually gathered to support a position or for some sort of decision-making purpose.
- Statistical data are obtained either by counting or measuring.
- Raw data are collected data that have not been organised numerically. Statistical data are obtained either by counting or measuring

Reflection

- What are some of the experiences (i.e., cognitive, psychomotor, and affective) I went through at the basic/secondary/tertiary level(s)? How have these experiences prepared me to achieve the school curricula aims, values and aspirations?
  How have my experiences in this training session prepared me to be a better classroom practitioner? Which specific examples can I draw from the course to support my position?

Discussion

- How has this session equipped you to be a better classroom practitioner?

SESSION 2: ORGANISATION OF DATA

When data has been collected, it is often essential to set it out in a tabular form. This is done in either of the following two ways: ungrouped form and grouped form.

Learning outcomes

By the end of this session, you will be able to:
1. Identify organised a raw data in ungrouped or grouped frequency distribution tables
2. Construct ungrouped or grouped frequency distribution tables from a given raw data

1 Identifying organised a raw data in ungrouped or grouped frequency distribution tables

Raw data gathered after collection, needs to be organised. Raw data are organised in frequency distribution tables. These tables are either ungrouped or grouped into categories. The frequency distribution tables are shown in classes with their corresponding frequencies. Examples of ungrouped and grouped frequency distribution tables are respectively shown below.

Ungrouped Frequency Distribution Table

Age(years)	Frequency
15	7
16	11
17	10
18	4
19	3
20	5
21	8

Grouped Frequency Distribution Table

Height (cm)	Frequency
145-149	6
150-154	9
155-159	7
160-164	4
165-169	3
170-174	5
175-179	8

2.2 Constructing frequency distribution tables

At the Senior High School, you were introduced to statistics. You were taught how to construct frequency distribution tables with simple data.

In this sub-session, we are going to revise with you how to construct ungrouped and grouped frequency distribution tables from a given raw data.

a. **Ungrouped frequency distribution table**

The data below are ages of (to the nearest year) of 35 students in Senior High School Form 1.

18	15	17	19	16	16	15
17	16	16	15	18	15	17
16	18	16	17	18	19	16
18	17	15	16	17	16	16
16	15	17	17	15	17	17

Data in this form is not very useful. The data can be illustrated in a tabular form so that useful information can be gained from it.

To construct this table, we should look through the raw data and identify the largest and smallest values.
The easiest way to tabulate it is by means of a tally chart. This is illustrated in the table below:

Age	**Tally**	**Frequency**
15		7
16		11
17		10
18		5
19		2

This table is an example of a **grouped frequency table**. This is because it shows the number of times (frequency) that the item of data occurs.

From the table, we can immediately see that majority of the students are 16 years old, two of the students are 19 years old and the ages range from 15 to 19 years.

b. **Grouped frequency distribution table**

If the data is very wide, then it would be easier to condense it somehow, to make it better to be interpreted. When dealing with data like this, we can group the values as shown in the data below:

The marks obtained by 50 students in a Mathematics examination are given below:

5	41	32	17	29	52	11	34	25	20
34	22	33	44	16	59	15	31	40	15
16	14	11	34	33	26	16	46	15	13
35	8	39	27	46	43	50	5	32	8
24	13	18	52	19	24	27	19	9	52

From the data above, the following groups could be used: 0-9, 10-19, 20-29 and so on.

Marks	Tally	Frequency
0-9		5
10-19		15
20-29		8
30-39		11
40-49		6
50-50		5
		50

If we compare this table to the raw data, we will notice that it is easier to get some useful information from the table than from the raw data.
For example, we can see that majority of the students scored marks between the intervals (10-19).

With the grouped frequency table, some useful information is lost. The table does not give the exact mark for each student.

Some new words to learn about grouped frequency tables are: **class interval, class limits, class boundaries, class size (class width) and class mark of a class interval.**

- The interval 21 – 30 defining the class in a grouped data is call a class interval.
- The end numbers, 21 and 30 are called class limits.
- 21 is the lower-class limit and 30 is the upper-class limit.
- Class boundaries which are sometimes used to symbolise classes are obtained by adding the upper limit of a class to the lower limit of the next class and dividing the result by 2.
- For example, in the table we have the following classes: 21 – 30, 31 – 40, 41 – 50 etc
- The class boundaries are thus: 20.5 – 30.5, 30.5 – 40.5, 405 – 50.5 etc respectively.
- The class size or width of a class interval refers to the difference between the upper boundary and he lower boundary.
- It is also referred to as class size, class width or class length. Class size of the class limits in the grouped data above is 10.
- The class mark also referred to as class midpoint of a class interval is the sum of the lower and the upper boundary divided by 2.
- Others can also be obtained by adding on successively the class width of the first-class interval.
- From the above data, the class marks are 5.5, 15.5, 25.5, 35.5, etc respectively.

Task: Investigate how one can obtain the class marks from the upper- or lower-class boundaries and the class width

Key ideas/core points

- Raw data are organised in frequency distribution tables.
- The frequency distribution tables are either ungrouped or grouped into categories.
- Class boundaries which are sometimes used to symbolise classes are obtained by adding the upper limit of a class to the lower limit of the next class and dividing the result by 2.
- Some new words to learn about grouped frequency tables are: **class interval, class limits, class boundaries, class size (class width) and class mark of a class interval.**
- The class size or width of a class interval refers to the difference between the upper boundary and he lower boundary.
- The class mark also referred to as class midpoint of a class interval is the sum of the lower and the upper boundary divided by 2.

SESSION 3: DATA REPRESENTATION

We shall in this session attempt to show representation of a statistical data using the following: Pie Chart, Pictogram, Stem and Leaf Plots, Histogram.

Learning outcomes

By the end of this session, you will be able to represent statistical data in the form of:

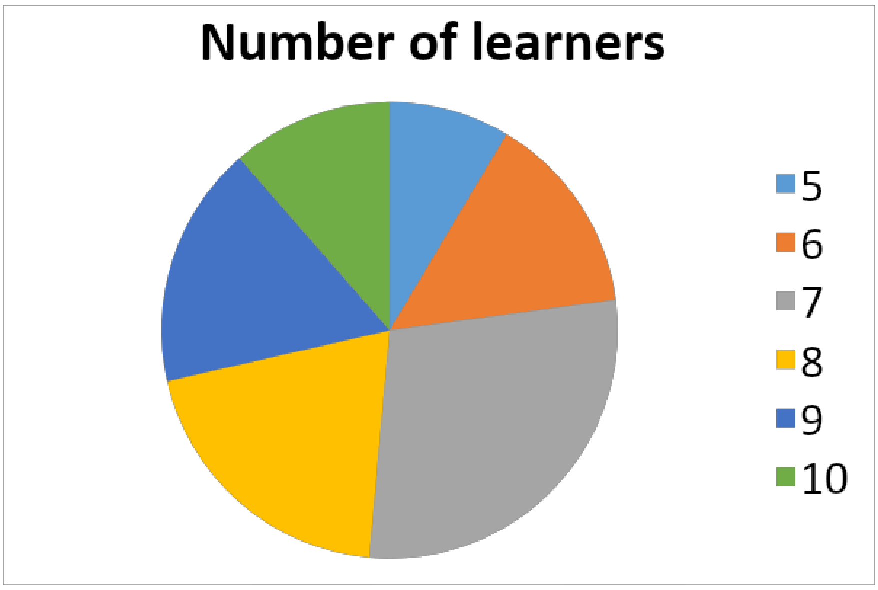
1. pie charts
2. pictograms/pictographs
3. stem and leaf plots
4. histograms

3.1 Representing statistical data by pie charts

A pie chart is a circular graph divided into sectors, each sector representing a different item of the data. The angle of each sector is proportional to the value of the part of the data it represents.

Below is an example of a pie chart

Pie chart showing marks obtained by learners in a mathematics class test



The following are steps to follow when drawing a pie chart:

i. Complete a table of angle representation each subject in the data by going the steps below:
   - Add together the number of each item of the data.
   - Divide $360°$ by the total number of elements.
   - Find the sectorial angles of each element of the data by multiplying the angle for each element [angle of an item=itemTotal360o]

ii. Draw a reasonably - sized circle. Mark the centre O and draw a radius of the circle.
ii. Use a protractor to divide the circle into sectors, using the angles obtained in (i).
ii. Label each sector accordingly.

Example 1

The table below shows the number of students who scored more than 80% in listed subjects.

Subject	Number of students
Biology	26
Physics	30
Chemistry	32
French	38
Geography	24
History	30

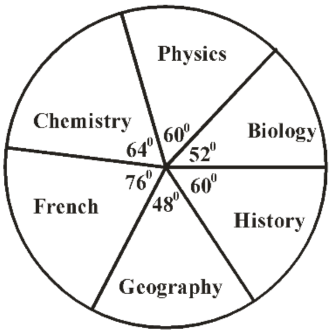
Draw a table of angles for the distribution.

Solution

Subject	Number of Students	Sectorial angles
Biology	26	$\frac{26}{180} \times 360^2 = 26 \times 2°$ =52o
Physics	30	30×2=60°
Chemistry	32	32×2=64°

French	38	38×2=78°
Geography	24	24×2=48°
History	30	30×2=60°
	180	360°

Below is a drawn pie chart for the distribution.



Key ideas/core points

- A pie chart is a circular graph divided into sectors, each sector representing a different item of the data.
- The angle of each sector is proportional to the value of the part of the data it represents

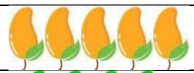
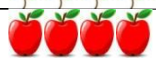
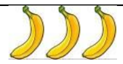
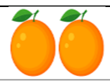
3.2 Representing statistical data by pictogram/pictograph

A pictogram is a chart that uses pictures or symbols to represent data. Pictograms are set out in the same way as bar charts, but instead of bars they use columns of pictures to show the numbers involved. For instance, members in the class may be asked to find out about favourite crisps, cakes, animals or colours of the students in their class or another class. Usually, the students record this information on a class list and then put it onto a **tally chart**. Later this information is then converted into a pictogram.

In drawing a pictogram, it is very important that the symbols used must be simple and clear. The quantity each symbol represents should be given as a **key**.

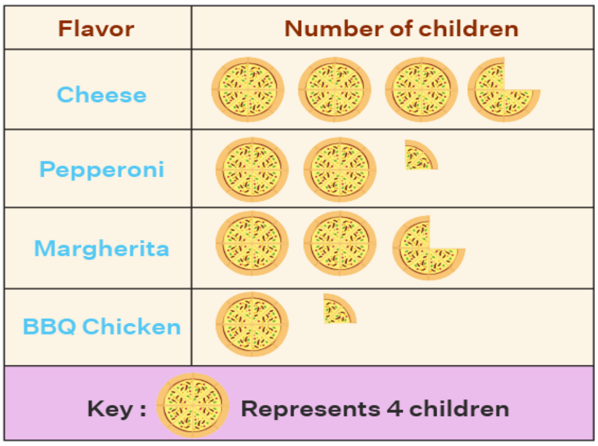
Example 1

The pictogram shows the survey on students who ate fruits in the particular semester

S/N	FRUIT	NUMBER OF STUDENTS
1	Mangoes	
2. 2	Apples	
3. 3	Bananas	
4. 4	Oranges	

In Statistics, pictographs are charts that are used to represent data using icons and images relevant to the data. A key is often included in a pictograph that indicates what each icon or image represents.

Example 2



Try and establish the similarities and differences between a pictogram and a pictograph. You will notice that all pictograms are pictographs but not all pictographs are pictograms.

3.3 Representing statistical data by stem-and-leaf plot

A stem-and-leaf plot is a special table where each data value is split into a "stem" (the first digit or digits) and a "leaf" (usually the last digit). Each stem can consist of any number of digits; but each leaf can have only a single digit.

A stem-and-leaf plot is a great way to organize data by the frequency. It is a great visual that also includes the data. So if needed, you can just take a look to get an idea of the spread of the data or you can use the values to calculate the mean, median or mode. A stem and leaf plot requires a key.

Let consider some examples

Example 1

The following shows the distribution of marks of students in an examination.

6 43 26 18 27
42 8 22 31 39
55 44 37 47 59
10 12 36 53 48

Make a **stem-and-leaf** plot of the marks above.

Solution

The stem-and-leaf of the marks:

Stem	Leaf
0	6, 8
1	8, 0, 2
2	6, 7, 2
3	1, 9, 7, 6
4	3, 2, 4, 7, 8
5	5, 9, 3

Key: 4 | 3 = 43

Note

a. Stem "0" Leaf "6" means 6
b. Stem "0" Leaf "8" means 8
c. Stem "1" Leaf "8" means 18
d. Stem "1" Leaf "0" means 10
e. Stem "1" Leaf "2" means 12
f. Stem "2" Leaf "6" means 26. . etc.

You are encouraged to finally represent the plot in such a way that the leaves will follow an increasing or decreasing order as shown below:

Stem	**Leaf**
0	6, 8
1	0, 2, 8
2	2, 6, 7
3	1, 6, 7, 9
4	2, 3, 4, 7, 8
5	3, 5, 9

Key: 5 | 3 = 53

Example 2

Make a **stem-and-leaf** plot of the following marks below:
552, 547, 578, 543, 559, 565, 544, 552

Solution

Stem	**Leaf**
54	3, 4, 7
55	2, 2
56	5
57	8
59	2

Key: 56 | 5 = 565

3.4 Representing statistical data by a histogram

In our previous level, we were able to identify situations and problems for data collection and appropriate methods for the collection of the data. We also learnt how to construct frequency tables for grouped and ungrouped data.

Histograms are just like bar charts but there are no spaces between the bars. Histograms are drawn to represent frequency distribution of continuous data. A histogram is a graph that shows frequencies of items of data. It is drawn using vertical bars.

These bars must be **of uniform width** (or equal width). The width of each bar **is equivalent to the class interval of the class** it represents. The area covered by the bar is proportional to the total frequency of the items in the class it represents.

How to Draw a Histogram
In drawing a histogram, the following steps should be followed:

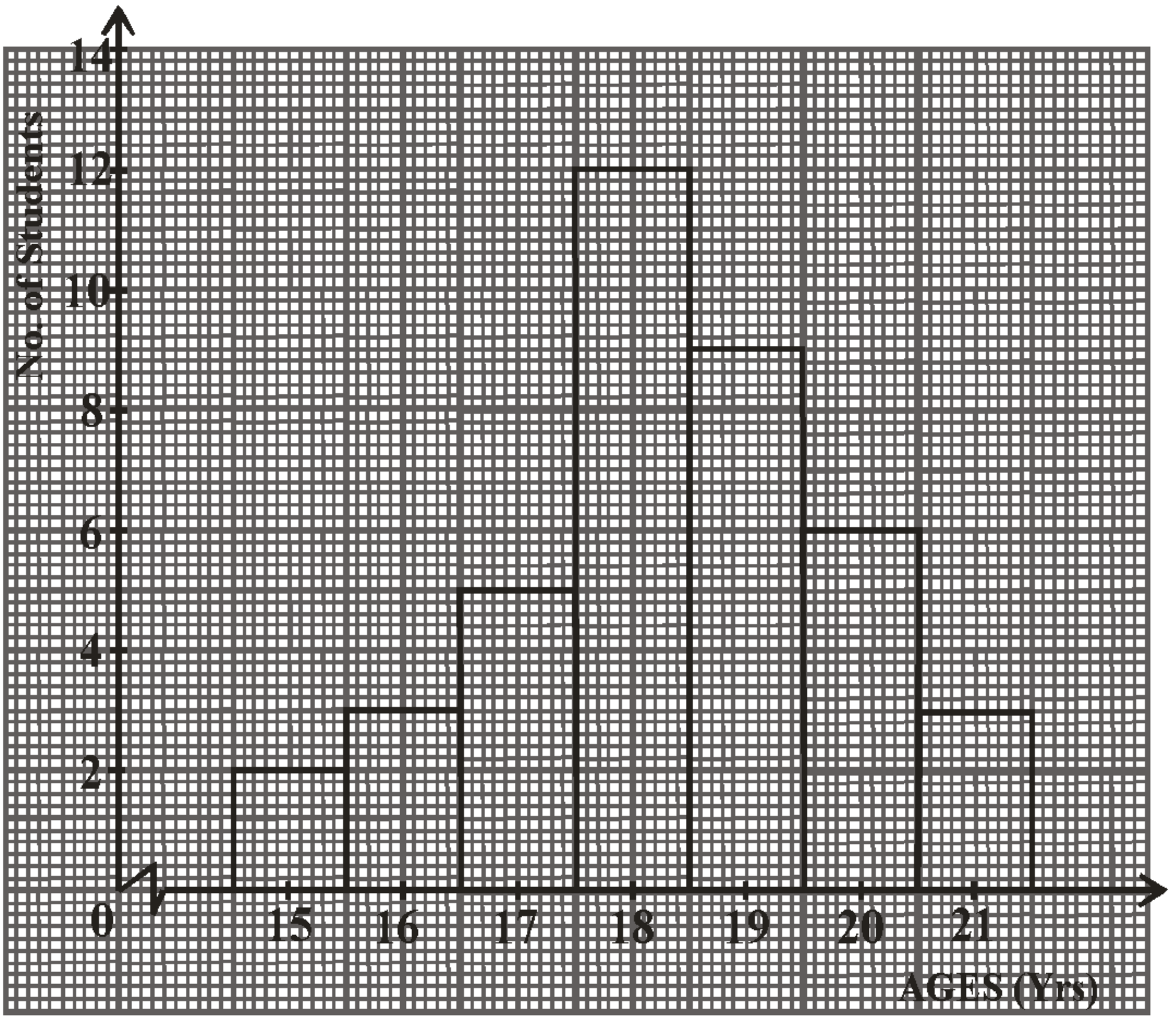
i. Choose a suitable scale for both axes;
ii. Draw two perpendicular axes. Label the axes. The vertical axis for the frequencies and the horizontal for the given data, for example ages (years), marks (%) and so on.
iii. Use two methods in which the bars are drawn:
    a. Use the class boundaries. Draw bars using the upper- and lower-class boundaries.
    b. Use of the class marks. The class marks should appear at the centres of the bars and each bar should have the same width as the class width. The ends of each bar should coincide with the class boundaries.
iv. Histograms should be drawn on graph sheets. The examples below show how histograms are drawn:

Example 1
The table below shows the ages of students in a class.

Ages (years)	15	16	17	18	19	20	21
No. of students	2	3	5	12	9	6	3

Draw a histogram for the distribution.

Solution



Note

1. An axis of the form indicates that the graph does not start from zero on the horizontal axis. The vertical axis should start from zero.
2. The ages (years) are at the centres of the bars.

Example 2

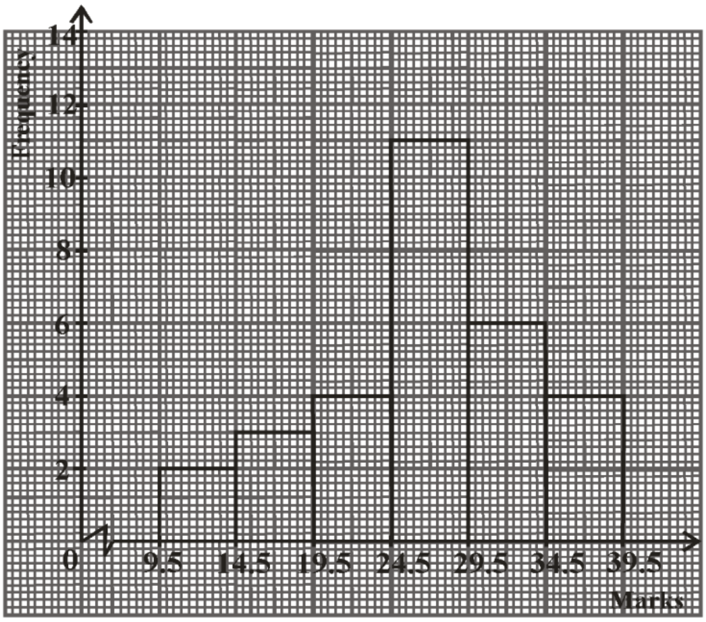
The table below shows the marks of some students in a test.

Marks	10-14	15-19	20-24	25-29	30-34	35-39
Frequency	2	3	4	11	6	4

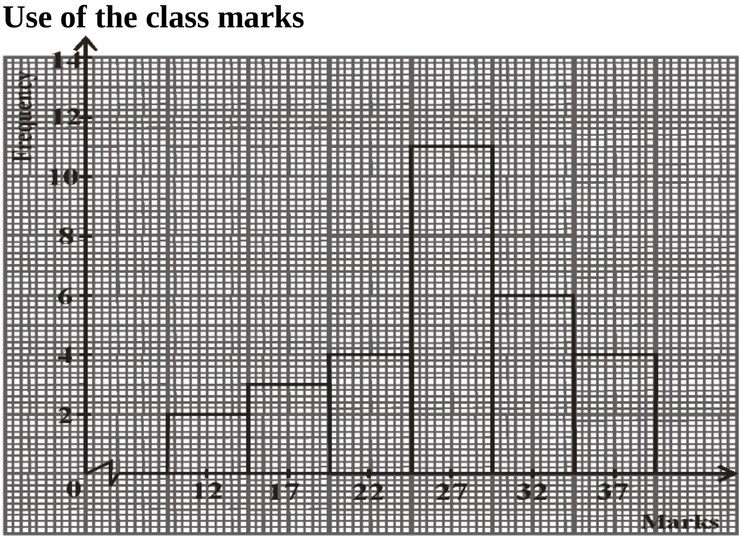
Draw a histogram for the distribution.

Solution

Use of class boundaries. Class mark for 10-14 is 12.



Use of the class marks



Key ideas/core points

- A pictogram is a chart that uses pictures or symbols to represent data.
- A stem-and-leaf plot is a special table where each data value is split into a "stem" (the first digit or digits) and a "leaf" (usually the last digit).
- Each stem can consist of any number of digits; but each leaf can have only a single digit.
- Histograms are drawn to represent frequency distribution of continuous data.
- A histogram is a graph that shows frequencies of items of data. It is drawn using vertical bars.
- A stem and leaf plot requires a key.

UNIT 4: STATISTICS II

Statistics is also about data interpretation. This can be achieved having knowledge about averages (central tendencies) and spread of the data about these averages (measures of dispersions). Each of these measures describes a different indication of the typical or central value in the distribution. In this unit we shall discuss central tendencies and measures of dispersions.

Learning outcomes

By the end of this unit, you will be able to:

1. find the measures of central tendency in the form of mean, median and mode from a given distribution.
2. calculate the measures of dispersion which are the range, variance and standard deviation

SESSION 1 FINDING THE MEASURES OF CENTRAL TENDENCY IN THE FORM OF MEAN, MEDIAN AND MODE FROM A GIVEN DISTRIBUTION

There are three main measures of central tendency: **the mode, the median and the mean**.
There are three main measures of central tendency: the mode, the median and the mean.
Each of these measures describes a different indication of the typical or central value in the distribution.

Learning outcomes

By the end of this session, you will be able to, from a given data, compute:

1. the mean
2. the median
3. the mode

1.1 Finding the mean of a data

We shall tackle this sub-session in three parts:

a. Finding the mean of a raw data
b. Finding the mean of an ungrouped data
c. Finding the mean of a grouped data

a. Finding the mean of a raw data

The mean of a set of numbers is the sum of all the numbers in the data divided by the total number of items. The mean is the measure of position commonly known as the **average.**

The mean, x, of *n* number x1, x2, x2, x3, ⋯, xn is given by

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

This can be written as $\bar{x} = \frac{\sum x}{n}$.

Example 1
Find the mean of 37, 38, 34, 29, 31, 35, 30, 36, 32 and 37.

Solution

$$\bar{x} = \frac{\sum x}{n}.$$

$$\bar{x} = \frac{37+38+34+29+31+35+30+36+32+37}{10}$$

$$\bar{x} = \frac{339}{10}$$

$$\bar{x} = 33.9$$

Example 2
Find the mean of 27, 21, 24, 23, 15, 18 and 12.

Solution
27, 21, 24, 23, 15, 18 and 12.

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$\bar{x} = \frac{27+21+24+23+15+18+12}{7}$$

$$\bar{x} = \frac{140}{7}$$

$$\bar{x} = 20$$

Example 3
The average of five numbers 4, 10, 24, *x* and 16 is 13. Find the value of *x*.

Solution
Since the average of 5 numbers is 13;
The sum of five numbers is $5 \times 13 = 65$

$$4+10+24+x+16+13 = 65$$

$$54 + x = 65$$

$$x = 65 - 54$$

$$x = 11$$

b. Finding the mean for ungrouped data

The method above can be applied to tabulated data. We will represent the attribute with *x* and the frequency *by f*. Consider the table below:

x	0	1	2	3	4	5
f	2	4	6	8	7	3

If we take the score 1 with frequency of 4, it means that 1 appears 4 times, their total is 1+1+1+1=4 which is the same as 1×4=4.
Similarly, the total score of 2 with frequency of 6 is 12, i.e. 2×6=12.
This can be done for all the scores.

Therefore, to find the mean, a column should be created for *fx*.
(i.e. frequency multiplied by the scores as shown in the table below).

x	f	fx
0	2	0×2=0
1	4	1×4=4
2	6	2×6=12
3	8	3×8=24
4	7	4×7=28
5	3	5×3=15
	$\sum f = 30$	$\sum fx = 83$

The mean, $\bar{x}$:

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$\Rightarrow \quad \bar{x} = \frac{83}{30}$$

$$\Rightarrow \quad \bar{x} = 2.766$$

$$\Rightarrow \quad \bar{x} = 2.8$$

Hence, for a frequency distribution, the mean is $\bar{x} = \dfrac{\sum fx}{\sum f}$.

c. Finding the mean for a grouped data

If we want to calculate the mean for grouped data, then we use the class mark to represent the various values in the group. The procedure is the same as calculating the mean for ungrouped data.

The mean, $\bar{x} = \dfrac{\sum fx}{\sum f}$

This is illustrated in the example below:

Example 4

The following are the masses (kg) of some students in a certain Senior High School class. Calculate the mean mass of the class to one decimal place.

Mass (kg)	45-49	50-54	55-59	60-64	65-69	70-74	75-79
Frequency	3	5	8	10	7	1	1

Solution

Construct a new table with **two** additional columns for class mark and *fx* as shown below:

Mass (kg)	**Class mark**	**Frequency**	***fx***
45-49	47	3	141
50-54	52	5	260
55-59	57	8	456
60-64	62	7	620
65-69	67	7	469
70-74	72	1	72
75-79	77	1	77
		f=35	fx=2095

The mean , $\bar{x} = \frac{\sum fx}{\sum f}$

$\Rightarrow \bar{x} = \frac{2095}{35}$

$\Rightarrow \bar{x} = 59.857\text{kg}$

$\therefore$ The mean is 59.9 kg

1.2 Finding the median of a data

The median of a set of numbers is the middle number or the arithmetic mean of the two middle values, if they are arranged in increasing or decreasing order of magnitude.

We shall consider finding the median for (a) an ungrouped and (b) a grouped data.

a. **Median for an ungrouped**

To find the median value for a set of discrete variables, for an ungrouped data, follow the steps below.

- Arrange the distribution in order of magnitude i.e. $x_1, x_2, x_3, \ldots, x_n$.
- If N (number of data items is odd, then the median is the middle value. i.e. $\text{Median} = \frac{1}{2}(N+1)^{th}$ item.

Example 5

Find the median mark of the following data: 1, 3, 5, 2, 9, 8, 11, 13, 14, 15, 17.

Solution

Ordering the data items (numbers) we obtain:

1, 2, 3, 5, 8, **9**, 11, 13, 14, 15,17 , which are $N = 11$ data items (numbers).

$\text{Median} = \frac{1}{2}(N+1)^{th}$ data item.

Median $= \frac{1}{2}(11+1)^{th}$ data item.

Median $= \left(\frac{12}{2}\right)^{th}$ data item.

Median $= 6\text{th}$ data item.

$\therefore$ The median mark is 9 .

Example 6

Find the median of the following set of numbers
27, 21, 24, 23, 15, 18 and 12.

Solution

Ordering data we have:

12, 15, 18, **21**, 23, 24, 27. Since $N = 7$ which is odd.

$\Rightarrow$ $\text{Median} = \frac{1}{2}(N+1)^{th}$ data item.

$\Rightarrow$ $\text{Median} = \frac{1}{2}(7+1)^{th} = 4^{th}$ data item.

$\therefore$ The median mark is 21.

If *N* is even, then the median is the arithmetic mean of the middle two items.

i.e the $\frac{1}{2}(N)^{th}$ and $\left(\frac{1}{2}N+1\right)^{th}$ items or $\frac{1}{2}(N)^{th}$ and the item after it.

Thus, $\text{Median} = \dfrac{\frac{1}{2}(N)^{th} + \left(\frac{1}{2}N+1\right)^{th}}{2}$ items

Example 7 Find the median from the data:
2, 1, 7, 5, 9, 8

Solution
Arranging the data in ascending order of magnitude, we obtain:
1, 2, **5**,$\downarrow$ **7**, 8, 9. Since $N = 6$ which is even.

$$\text{Median} = \frac{\frac{1}{2}(N)^{th} + \left(\frac{1}{2}N+1\right)^{th}}{2} \text{ items}$$

$$\text{Median} = \frac{\frac{1}{2}(6)^{th} + \left(\frac{1}{2}(6)+1\right)^{th}}{2} \text{ items}$$

$$\text{Median} = \left(\frac{3^{rd} + 4^{th}}{2}\right) \text{ items}$$

$$\text{Median} = \frac{5+7}{2}$$

$$\text{Median} = \frac{12}{2}$$

$$\text{Median} = 6$$

Example 8

Find the median mark of the following data: 3, 5, 2, 9, 8, 11, 13, 14.

Solution

2, 3, 5, **8**, $\downarrow$ **9**, 11, 13, 14. Since $N = 8$ which is even.

$$\text{Median} = \left(\frac{\frac{1}{2}(N)^{th} + \left(\frac{1}{2}N+1\right)}{2} \right)^{th} \text{ items}$$

$$\text{Median} = \left[\frac{\frac{1}{2}(8)^{th} + \left(\frac{1}{2}(8)+1\right)}{2} \right]^{th} \text{ items}$$

$$\text{Median} = \left(\frac{4^{th} + 5^{th}}{2} \right) \text{ items}$$

Median mark $= \frac{8+9}{2}$

Median mark $= \frac{17}{2}$

Median mark $= 8.5$ marks

From the data above, the median mark is 8.5.

a. **Median for a grouped data**

Median=l+hn2-cf

Where,

l = Lower boundary of the median class
c= Cumulative frequency

f = Frequency f median class
h = Class size
n = Number of even observations
Median class = Class where n/2 lies

But for n odd observations, Median=l+h (n+1)2-cf

Example 9. Find the median of the distribution below:

Class	Frequency	Cumulative Frequency
1 - 10	1	1
11- 20	3	4
21 - 30	6	10
31 - 40	4	14

41 - 50	**2**	**16**
	n=16	

Solution.

Data
- the median class =n2
- Thus the median group is 21 - 30
- *l* = 20.5
- c = 4
- f = 6
- h = 9
- n = 16
- Median class = Class where n/2 lies

Median=20.5+108-46

Median=20.5+1046

Median=20.5+108-46

Median=27.16

1.3 Finding the median of a data

The mode of a set of numbers is the value which occurs with the highest frequency or the mode is the value that occurs most frequently.

We consider finding the mode of (a) an ungrouped data and that of (b) a grouped data.

a. Mode of an ungrouped data

Example 1
Find the mode of the following data: 2, 4, 5, 6, 7, 7, 7, 8, 9.

Solution

2, 4, 5, 6, **7**, **7**, **7**, 8, 9

Therefore the mode is 7.

Example 2
Find the mode of the following data
i. 23, 12, 15, 12, 21, 18, 21, 23 and 21.
ii. 2, 3, 2, 3, 4, 3, 5
iii. 3, 2, 2, 4, 3, 5, 4, 2, 4, 2, 3

Solution

i. 23, 12, 15, 12, **21**,18, **21**, 23 and **21**.
∴ The mode is 21.

ii. 2, **3**, 2, **3**, 4, **3**, 5
∴ The mode is 3.

iii. 3, **2**, **2**, 4, 3, 5, 4, **2**, 4, **2**, 3
∴ The mode is 2.

b. **Mode of a grouped data**

- The mode for grouped data or ungrouped data can be calculated using the mode formulas given below,
- Mode for grouped data:

Mode=L+hfm-f1fm-f1+fm-f2

Where:

- *L* is the lower boundary of the modal class
- *h* is the size of the class interval
- f_m is the frequency of the modal class
- f_1 is the frequency of the class preceding the modal class
- f_2 is the frequency of the class succeeding the modal class

Example 2

Find the mode of the data shown below:

Marks	Frequency
1 – 10	**1**
11- 20	**3**
21 – 30	**6**
31 – 40	**4**
41 – 50	**2**

Solution

From the table,

- Modal class = 21 – 30 (Class with the highest frequency.
- Lower boundary of the modal class (*L*) = 20.5
- Class size of the class interval (*h*) = 9
- frequency of the modal class (f_m) = 6
- f_1, frequency of the class preceding the modal class = 3
- f_2, frequency of the class succeeding the modal class = 4

Thus, Mode=20.5+96-36-3+6-4)
Mode=20.5+935
Mode=20.5+935
Mode=25.9

Key ideas/core points

- There are three main measures of central tendency: **the mode, the median and the mean**.
- The mean, x, of n items (marks/scores) x1, x2, x2, x3, ⋯, xn with corresponding frequencies f1, f2, f3, f4, ⋯, fnis given by $\bar{x} = \frac{\sum fx}{\sum f}$ where, n=f.
- For a grouped data, x is the class mark of the distribution.
- For ungrouped data set, the median of a set of numbers is the middle number or the arithmetic mean of the two middle values, **if they are arranged in increasing or decreasing order of magnitude**.
- Median for a grouped data is computed as:

Median=l+hn2-cf
Where,
l = Lower boundary of the median class
c= Cumulative frequency
f = Frequency f median class
h = Class size
n = Number of even observations/ (n+1) for number of odd observations
Median class = Class where n/2 lies

- For ungrouped data set, the mode of a set of numbers is the value which occurs with the highest frequency, or the mode is the value that occurs most frequently.

For grouped data set, the mode is computed using the formula
Mode=L+hfm-f1fm-f1+fm-f2
Where:

L is the lower boudary of the modal class
h is the size of the class interval
f_m is the frequency of the modal class
f_1 is the frequency of the class preceding the modal class
f_2 is the frequency of the class succeeding the modal class

SESSION 2: MEASURES OF DISPERSION

A measure of dispersion is the study of the ways in which a set of data is arranged around it centre. That is the way the data is dispersed around its centre (e.g. the mean). The most important measures of dispersion are the range, inter-quartile range, semi-inter-quartile range, standard deviation and variance.

Learning outcomes

By the end of this session, you will be able to:
1. **Find the range**
2. **The variance and standard deviation**

2.1 Finding the range of a data.

The range is the difference between the largest and the smallest observation in the data. The prime advantage of this measure of dispersion is that it is easy to calculate.

$\text{Range} = X_{\max} - X_{\min}$

Example 1

Find the range of the given data:1, 3,5, 6, 7.

Solution

Range is $7-1=6$

2.2 Finding the variance and the standard deviation of a distribution

a. **Variance**

Variance is the expected value of the squared variation of a random variable from its mean value, in probability and statistics.

Informally, variance estimates how far a set of numbers (random) are spread out from their mean value. The value of variance is equal to the square of standard deviation, which is another central tool.

Variance is symbolically represented by $\boldsymbol{\sigma^2}$, $\mathbf{s^2}$, or **Var(*X*).**

b. **Standard deviation**

Standard deviation (SD) is the most commonly used measure of dispersion. It is a measure of spread of data about the mean. SD is the square root of sum of squared deviation from the mean divided by the number of observations. In other way round, SD is the square root of variance.

We shall consider how to find the standard deviation of an ungrouped data and a grouped distribution.

- **Finding the standard deviation of ungrouped distribution**

  If $x_1, x_2, \ldots, x_n$ occur with frequencies $f_1, f_2, \ldots, f_k$ respectively, then the standard deviation is given

by: (a) $S=\sqrt{\dfrac{\sum(x-\bar{x})^2}{N}}$, where $N=\sum f$ or (b) $S=\sqrt{\dfrac{\Sigma x^2}{N}-\left(\dfrac{\Sigma x}{N}\right)^2}$ or (c) $S=\sqrt{\dfrac{\Sigma d^2}{N}-\left(\dfrac{\Sigma d}{N}\right)^2}$

Example 2

Find the standard deviation from the following distribution: 2, 4, 6, 8, 10, 12.

Solution

x	$x-\bar{x}$	$(x-\bar{x})^2$
2	-5	25
4	-3	9
6	-1	1
8	1	1
10	3	9

12	5	25
$N=6$		$\Sigma(x-\bar{x})^2=70$

(i) $\bar{x}=\dfrac{2+4+6+8+10+12}{6}=\dfrac{42}{6}=7$

(ii) $S=\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{N}}=\sqrt{\dfrac{70}{6}}=\sqrt{11.6667}=3.4157$

$\Rightarrow S=\sqrt{\dfrac{855}{5}-\left(\dfrac{65}{5}\right)^2}$ $\Rightarrow S=\sqrt{171-(13)^2}$ $\Rightarrow S=\sqrt{171-169}$ $\Rightarrow S=\sqrt{2}$ $\Rightarrow S=1.4142$ $\Rightarrow S=1.4$

- **Finding the standard deviation of a grouped distribution**

For the grouped data, standard deviation is given by: $S=\sqrt{\dfrac{\sum fx^2}{\sum f}-\left(\dfrac{\sum fx}{\sum f}\right)^2}$

Example 3

Calculate the standard deviation from the frequency distribution:

Class Intervals	1-5	6-10	11-15	16-20	21-25
Frequency	4	6	11	6	3

Solution

From the formula:

$$S=\sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

We generate like the one shown below:

Class Interval	f	Class Midpoint (x)	fx	x^2	fx^2
1-5	4	3	12	9	36
6-10	6	8	48	64	384
11-15	11	13	143	169	1859
16-20	6	18	108	324	1944
21-25	3	23	69	529	1587
	$\Sigma f=30$		$\sum fx=380$		$\sum fx^2=5810$

Thus,

$$= \sqrt{\frac{5810}{30} - \left(\frac{380}{30}\right)^2}$$
$$= \sqrt{193.6667 - 160.4444}$$
$$= \sqrt{33.2223} = 5.7639$$

We can calculate the variance therefore, by squaring the standard deviatiob.
Thus, Vx=33.22332=33.2233

UNIT 5: PROBABILITY I

Probability is an important aspect of statistics which possesses wide application in life. Some day-to-day words which express the idea of probability includes probable/improbable, chance, possibility/impossibility, likely/unlike, certainty/uncertainty, sure/unsure, and others you can think of.

Learning outcomes

By the end of the unit, the student will be able to explain the concept of:

1. simple outcome invents
2. compound outcome invents
3. mutually exclusive outcome events

SESSION 1: EXPLAINING THE CONCEPT OF SIMPLE OUTCOME EVENTS

We shall in this session discuss the simple events and their outcomes in probability. Each outcome event will be examined, and experiments involved in it, conducted to make it more meaningful.

Learning outcomes

By the end of this session, you will be able to:

1. identify some simple events
2. state the possible outcome of simple events

1.2 Identifying outcome events

A simple (or single) event is an event with a single outcome (only one "answer")

Experiments involving simple outcome events includes:

- Tossing of a coin
- Throwing of a die
- Playing of a game
- Drawing or picking or selecting from a lot
- Favourable or Possible Results

1.2 Stating the possible outcome of simple events

Experiment	**Possible outcomes**
Tossing of a coin	{Head (H), Tail (T)}
Throwing of a die	{1, 2, 3, 4, 5, 6}
Playing of a game	{win, loss, draw}
Drawing or picking or selecting from a lot.	Depends
Choosing one answer from a multiple choice question with 5 options	{A, B, C, D, E}

Key Ideas

- A simple (or single) outcome event is an event with a single outcome (only one "answer")
- Examples: Tossing of a coin with possible outcomes {Head (H), Tail (T)} and throwing of a die with possible outcomes {1, 2, 3, 4, 5, 6}

SESSION 2: IDENTIFYING AND STATING COMPOUND OUTCOME EVENTS

A compound event is the combination of two or more simple events (with two or more outcomes). If an event has more than one sample point, it is termed as a compound event. The compound events are a little more complex than simple events.

Learning outcomes

By the end of this session, you will be able to:

1. identify compound events
2. state the outcomes of compound events

1. Identifying compound outcome events

Compound events include:

a. Tossing a coin twice (tossing two coins once)
b. Throwing of a die, twice (throwing two coins, once)
c. Tossing of a coin and throwing a die, once
d. Playing of a game and tossing a coin
e. Drawing red and a blue from a lot

2. Stating possible outcome of compound events experiments

a. Tossing a coin twice (tossing two coins once)

Second Coin ↓	First Coin → H	T
H	(H, H)	(H, T)
T	(T, H)	(T, T)

Thus, outcome event is {(H, H), (H, T), (T, H), (T, T)}

b. Throwing of a die, twice (throwing two coins, once)

White Die	Black Die						
		1	2	3	4	5	5
	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 4)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Thus, the possible outcome of the event is {(1, 1), (1, 2), (1, 3), (1, 5) …, (6, 6)}

c. Tossing of a coin and throwing a die, once

A Coin	A die					
1	2	3	4	5	6	
H	(H, 1)	(H, 2)	(H, 3)	(H, 4)	(H, 5)	(H, 6)
T	(T, 1)	(T, 2)	(T, 3)	(T, 4)	(T, 5)	(T, 6)

Thus, the possible outcome of the event is {(H, 1), (H, 2), (H, 3), …, (T, 6)}

The words "**or**" and "**and**" are often used in probability. Events combined are called **compound events**. "***or***" and "***and***" correspond to $\cup$ (union) and $\cap$ (intersection) respectively with respect to sets.

Example

(i) E_1 **or** E_2 or both gives $E_1 \cup E_2$

(ii) E_1 ***and*** E_2 gives $E_1 \cap E_2$.

Key Ideas

- A compound event is the combination of two or more simple events (with two or more outcomes).
- Examples: Tossing a coin twice (tossing two coins once) with possible outcomes {(H, H), (H, T), (T, H), (T, T)}

SESSION 3: IDENTIFYING AND STATING THE OUTCOMES OF MUTUALLY EXCLUSIVE EVENTS

Two events are mutually exclusive if they cannot occur at the same time. The simplest example of mutually exclusive events is a coin toss. A tossed coin outcome can be either head or tails, but both outcomes cannot occur simultaneously. It is commonly used to describe a situation where the occurrence of one outcome supersedes the other. For example, war and peace cannot coexist at the same time. This makes them mutually exclusive. Mutually exclusive events are events that can't both happen. For a basic example, consider the rolling of dice. You cannot roll both a five and a three simultaneously on a single die.

Learning outcomes

By the end of this session, you will be able to:

1. identify and state the outcomes of mutually exclusive events

3.1 Identifying mutually exclusive events

Examples

Examples include:

- right- and left-hand turns,

- even and odd numbers on a die,
- winning and losing a game,
- running and walking.
- When tossing a coin, the event of getting head and tail are mutually exclusive events. Because the probability of getting head and tail simultaneously is 0.
- In a six-sided die, the events “2” and “5” are mutually exclusive events. We cannot get both events 2 and 5 at the same time when we threw one die.
- In a deck of 52 cards, drawing a red card and drawing a club are mutually exclusive events because all the clubs are black.

For example if a coin is tossed and a die is thrown they cannot have any common point of intersection. Therefore, their point of intersection would be a null set. It implies that the two events cannot occur together.

If E_1 and E_2 are events such that $E_1 \cap E_2 = \phi$, then we say that E_1 and E_2 are mutually exclusive events:

There would be no intersection since that gives a null set. That is E_1 and E_2 are disjoint sets.

Key Ideas

- Mutually exclusive events cannot have any common point of intersection.
- The point of intersection of mutually exclusive event would be a null set.
- Two mutually exclusive events cannot occur together.
- Mutually exclusive events are called disjoint events.