Module for B.Ed Early Childhood Education Programme

EBS322SW:
METHODS OF TEACHING PRIMARY SCHOOL MATHEMATICS

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UNIT 1: CONCEPT FORMATION AND HOW LEARNERS LEARN MATHEMATICS

Welcome to the first unit of the course Methods of Teaching Primary School Mathematics. In this unit, we will discuss what a mathematical concept is, primary and secondary concepts and how learners form concepts. We shall also look at how learners learn mathematics and the implications for teaching mathematics in primary schools.

Learning outcome(s)

By the end of the unit, the participant will be able to:

1. explain a mathematical concept
2. identify primary and secondary concepts
3. explain how learners form concepts

SESSION 1: EXPLAINING A MATHEMATICAL CONCEPT

In mathematics, we learn about numbers, shapes, measurement and many other things or objects. To understand these things we first need to know what each of them is. We need to know for example what the number three, is; if a shape is a triangle or not, and so on. Once we know what these things are and how to identify them, we can then learn how they relate to other things and how we can use them. When we are able to tell what an object is and how to identify it, we have learnt the concept of that object.

Learning outcome(s)

By the end of this session, you should be able to;

1. explain what a mathematical concept is;
2. explain how we can abstract from a number of examples or instances what they all have in common.

1.1 Explaining mathematical concept

Richard Skemp, a mathematics teacher and psychologist, describes a concept as ‘the mental object, which results when we abstract from a number of examples or instances something which they all have in common’. Thus, a concept is a mental representation of common properties or regularities abstracted from isolated experiences.

1.2 Explaining how we can abstract from a number of examples or instances what they all have in common

When we focus our attention on the common property of a set of objects, we are abstracting or ‘pulling out’ or ‘extracting’ this common property. The result of this mental activity is a concept. This process of abstraction involves becoming aware of something in common among a number of experiences. Concept is some kind of lasting change, the result of abstracting which enables us to recognize new experiences as having similarities of an already formed class. For example, the concept ‘red’ is an awareness of something common among the experiences one has had. Without these experiences or similar ones, it would not have been possible for one to form the concept. By arranging for a learner to have these experiences, we make it possible for him or her to form the concept. From visual set of experiences, a learner abstracts the concept ‘red’. The words we use while indicating ‘red cap’, ‘red pen’ ‘that shirt is red’ enables the learner abstract what these descriptions have in common; that is, the word ‘red’. By associative learning, the learner attaches the concept to the word.

The mathematical concept ‘two’ is an awareness of something common among groups containing a pair of objects each. We can show a child variety of pairs of objects until he or she has assimilated the concept of ‘two’. The learner abstracts the ‘twoness’ property of the groups. In this way, though we cannot tell him or her the concept, we can cause it to form in his or her mind from a variety of examples, which have the concept in common.

Key Ideas

- a concept is a mental representation of common properties or regularities abstracted from isolated experiences.
- From visual set of experiences, a learner abstracts a concept

Self- Assessment Questions

1. Explain the term ‘concept’ in mathematics.
2. Give and explain three different examples of mathematical concepts you would introduce to Basic 3 pupils.

SESSION 2: IDENTIFYING PRIMARY AND SECONDARY CONCEPTS

Suppose we wish to convey the concept ‘number’ to a learner. It may not be helpful trying to communicate this by pointing. Instead, we would use words and say something like “two’ ‘three’ ‘four’, and so on, are all numbers.
In this session, we would discuss two kinds of concept – Primary and Secondary concepts.

Session learning outcome(s)
By the end of this session, you should be able to:
1. identify and explain what a primary concept is.
2. identify and explain what a secondary concept is.

2.1 Identifying and explaining a primary concepts
Primary concepts are concepts that we learn through our experiences or interaction with the environment. They are derived directly from our sensory and motor experiences of the outside world like the five senses of seeing, feeling, smelling, tasting and hearing. Examples of primary concepts are ‘red’, ‘green’, ‘yellow’, ‘blue’, ‘one’, ‘two’, ‘three’, ‘rectangle’, ‘triangle’, circle’, ‘heavy’, ‘sweet’, ‘hot’, ‘cold’, and so on.

The concept ‘three’ can be formed by seeing or experiencing many groups with three objects and recognizing their common property. Similarly, the concept ‘red’ can be formed when we see many red objects. Also, the concept ‘hot’ can be formed by touching hot objects.

2.2 Identifying and explaining a secondary concepts

Secondary concepts are derived from other concepts. They are built up by combining primary concepts. For example, by considering ‘one’, ‘two’, ‘three’, and so on, together we form the concept of ‘number’. Also, ‘red’, ‘green’, ‘yellow’, ‘blue’, and so on, together form the secondary concept ‘colour’. Examples of secondary concepts are number, colour, and shape. Multiplication is a secondary concept, being formed from that of addition.

Secondary concepts thus depend on other concepts and so can only be formed if the learner has already formed these other concepts.

Key Ideas

- Primary concepts are concepts that we learn through our experiences or interaction with the environment through the direct use of our sensory and motor experiences (seeing, feeling, smelling, tasting and hearing)
- Examples of primary concepts are ‘red’, ‘green’, ‘yellow’, ‘blue’, ‘one’
- Secondary concepts are derived from other concepts. They are built up by combining primary concepts.
- Examples of secondary concepts are number, colour, and shape

Self- Assessment Questions

1. Using three examples each distinguish between primary and secondary concepts.
2. Using two examples in each case, explain the terms ‘higher-order’ and ‘lower-order’ concepts.
3. Identify five concepts in different strands in mathematics which are primary concepts.
4. Identify five concepts in different strands in mathematics which are secondary concepts

SESSION 3: EXPLAINING HOW LEARNERS FORM CONCEPTS:

Session learning outcome(s)

By the end of this session, you will be able to:

1. Explain how learners form concepts

3.1 Explaining how learners form concepts

To form a concept, learners require a number of common experiences relating to the concept. Learners learn mathematics by abstracting concepts from concrete experiences. The language and symbols that name concepts are developed during or after concept formation.

Concept formation has to happen in the learner’s own mind, and teacher cannot do it for him or her. What the teacher can do is to help along the natural learning processes.

To avoid misconceptions about the procedural and conceptual knowledge of mathematics, it is important for us to know as teachers how learners form concepts. The following are various ways learners form concepts.

a. **Discrimination**
Learners interact with their immediate environment and begin to discriminate. That is, they manipulate materials, find out the objects bearing common properties and classify them into groups. Leaner's behaviour in recognizing objects is based on classification of their previous experiences and the fitting of their present experiences into one of these classes, that is, using past experiences to understand new ones. To classify is to collect together our experiences on the basis of their similarities.

b. **Similarities and differences**
Similarities and differences are used in concept formation. In this regard, learners continue to look for and describe the similarities and differences among the examples before them. From a learner's past involvements or experiences, he/she abstracts certain invariant properties, which persist in his/her memory longer.

c. **Abstraction**
Learners use abstraction in concept formation. Here, the learner begins to abstract the main or invariant properties of the examples given. An abstraction is a mental representation of a mathematical object. To 'abstract' is to extract what is common to a number of different situations and to disregard what is irrelevant. Thus, 'abstraction' is the process of formulating generalized concepts of common properties by disregarding the differences. For example, an individual learner is able to form the concept of 'three' by extracting the idea of 'threeness' from three books, three pens, three boys, three girls, three sticks, etc. The learner's ability to abstract the concept of 'threeness' does not in any way depend on the type of objects involved in the process. Also, the learner is not able to understand what is meant by 'number' until he/she has understood 'two' 'three', 'four' and other similar concepts. Here, the learner is able to understand the concept 'number' by abstracting from objects of 'one', 'two', 'three', 'four, and so on. 'Number' is an abstraction from a set of abstractions. The concept of 'addition of numbers' is an even higher abstraction than 'number'.

d. **Generalizations**
Finally, the learner draws some general conclusions from a number of experiences. To generalize is to claim that something is true. When we get learners to work at expressing their own generalizations, then mathematical thinking and learning is taking place. Learners begin to make generalizations when they begin to address the question 'Does this always work?'

Key Ideas

- To form a concept, learners require a number of common experiences relating to the concept
- Concept formation has to happen in the learner's own mind, and teacher cannot do it for him or her.

- Learners form concepts by discriminating, looking for similarities and differences, abstracting, and generalizing,

Self- Assessment Questions

1. Describe the following ways by which a learner forms concepts:
    a. discrimination
    b. identification of similarities and differences
    c. abstraction
    d. generalization
2. Describe two implications of concept formation for the mathematics teacher.

UNIT 2: TEACHING OPERATIONS ON WHOLE NUMBERS AND RATIONAL NUMBERS

Concepts of number begin early in a child's life and extend beyond being able to count objects or to recognize numerals. A good foundation in number concepts is important in the primary years because it is the basis for much of the work in mathematics that the child will be involved in throughout life in school and outside.
In this session we shall see how we can help learners learn to count objects and be able to recognize as well as write numerals. We shall also learn how we can guide learners to describe the positions of objects arranged in an order.

Learning outcome(s)

By the end of the unit, the participant will be able to:

1. Counting and gaining number sense (Cardinal, ordinal, and nominal numbers)
2. Define the basic arithmetic operation (addition, and subtraction, multiplication, and division and their properties)
3. Conceptual development of common fractions and basic operations on common fractions

SESSION 1 COUNTING AND GAINING NUMBER SENSE

Learners' earlier experiences with the objects in a group are helpful in their learning and understanding of number. When learners pair all the objects found in two given groups on one-to-one basis, they find out that there are as many objects in one group as in the other. We say that the groups have the same **number** of objects. Thus, we see that a number denotes the property which a group of equivalent sets have in common.

Session learning outcome(s)

By the end of this session, you will be able to help your learners to identify:

1. cardinal numbers,
2. ordinal numbers,
3. nominal numbers

1.1 Identifying cardinal numbers

The number property of a set which enables one to tell "how many" members there are in a set is known as a **cardinal number**. In other words, a cardinal number tells how many objects or members are in a set or group.

To tell the exact number of objects in each group, we need to guide the learners to count them. Often, we hear learners recite the number names in the right order quite easily. This is not the same as saying that they are able to count.

When we count, the purpose is to assign a number to a group of objects or persons. To count the objects in a group, the learner should be assisted to move each object in the group away from those not yet counted to a new place before he assigns a number name in the right order. The number name said gives the number of objects in the group in the new place. So, when the learner says 'one' he or she has one object put aside. He then returns to the group and moves another object away to the new place beside the first and says 'two'. He now has two objects at the new place; that is, he has counted two objects so far. He again returns to the group and moves another object away to the new place beside the first two and says 'three'. He repeats this activity until the last object or person is moved to the new place. The last number name mentioned gives the number of objects in the group.

This means that for a learner to be able to count efficiently he or she must be taught the number names in the conventional order, that is, 'one', 'two', 'three' and so on. Thus, when we count the objects in the group given in Fig 2.1, we have 'five' objects.

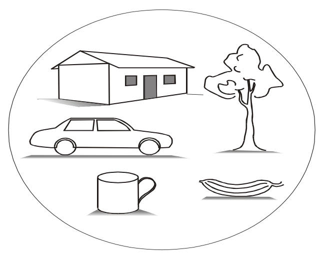


Fig. 2.1 Five objects

The learner can say then that the cardinal number or simply the number of objects in the group is five. It does not matter what order the learner counts the objects. He or she just wants to know how many there are.

Next, we should guide the learner to associate groups of objects with both the numerals or symbols and the number names. We also teach them how to write the numerals and then the number names. Most learners require a great deal of assistance in forming the shape of the numeral. We can give them much encouragement through practice with large shapes of the numerals on sand or on the ground. We can also prepare large numerals from clay for learners to examine and follow their outlines.

We should also engage learners in several activities and games by which they would come to understand cardinal number. Some activities are given below:

i. Boxes or trays like the one shown in Fig 2.2 are prepared. Learners are asked to place items in each box according to the numeral or the number name indicated on it.

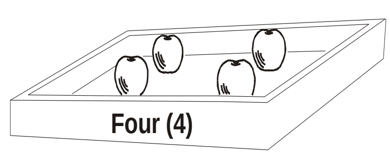


Fig 2.2

ii. The teacher and the pupils prepare a cardboard on which are fixed a group of say three buttons, three triangular cut-outs, and three postage stamps (Fig 2.3). These serve as sample groups of three objects. The numeral 3 and its name are written in the middle of the cardboard. Individual pupils are asked to place the number of counters indicated in the pocket attached to the cardboard. The teacher helps the learners to check these to ensure that the correct number of counters have been put in the pocket.

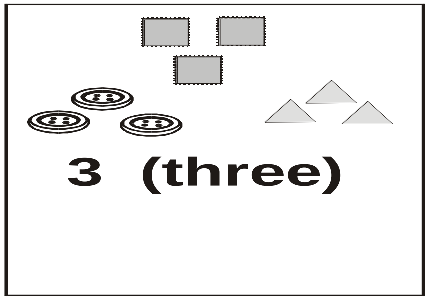


Fig. 2.3

We should note that it is difficult for learners to understand the number 'zero', since it is not a counting number. Learner's learning difficulty may arise because zero is the number property for the empty set and since they cannot see an empty set, they do not find it easy in relating the name zero to this special set. We can help learners understand the concept of zero by asking them to identify a group of pupils with two heads in their classroom or a group of pupils taller than the school block, and so on. They can see that in each case the group has no members. We then explain to them that the number of members in each group is zero, and the numeral is '0'.

We can help learners to learn the sequence of numbers by teaching them number rhymes and songs. The learners also have fun when they recite the rhymes and sing the songs while they perform actions that go with them. Some rhymes we can use to teach numbers to learners are given below:

One, two, three, four, five
Once I caught a fish alive
Six, seven, eight, nine, ten,
Then I let it go again.
Why did you let it go?
Because it bit my finger so
Which finger did it bite?
My little finger on the right.

One, two, buckle my shoe,
Three, four, shut the door,
Five, six, pickup sticks,
Seven, eight, lay them straight,
Nine, ten, a big fat hen.

1.2 Identifying ordinal numbers

We introduce ordinal numbers to learners after they have learnt to count accurately and have understood the notion of cardinal number. An ordinal number tells the position of an object or person in relation to other objects or persons. We explain to learners that we use ordinal numbers when we are interested in where an object or person is found in an orderly arrangement of objects or persons, usually in a row or column.

We should give learners the opportunity to determine the positions of objects or people arranged in a specified order. We should guide them to use the correct mathematical language to describe them. A learner points to a book in a pile and says, for example, "This is the first book from the top". Another mentions the name of, say, the second learner from the front seated in the third row from the teacher's table. We can also engage learners in activities such as racing to determine positions.

1.3 Identifying Nominal Numbers

Nominal numbers can be used to name an object or a person. For example, the number "2" can be used to identify the shirt used by a player in a football team. Also, a pupil in class may be given the number 9 and a class of pupils may be given the name class 4 for identification purposes. Draw your learners' attention to several of such a numbering system using more examples and lead them to conclude that nominal numbers are used for identification purposes and thus one can conclude that they are identification numbers. Make them realise that nominal numbers do not depict quantity. It is important to help learners distinguish between numbers used in the nominal sense and those used in the cardinal sense. Also, assist your learners to draw a clear distinction between numbers used in the ordinal sense and those that are used in the nominal sense.

Key Ideas

- A good foundation in number concepts is the basis for much of the work in mathematics that the child will be involved in throughout life in school and outside.
- A cardinal number tells how many objects or members are in a set or group.
- An ordinal number tells the position of an object or person in relation to other objects or persons.
- Nominal numbers can be used to name or identify an object or a person.
- Learners learn the sequencing of numbers by reciting number rhymes and singing number songs.

Self- Assessment Questions

Answer questions 1 and 2 by filling the blank spaces and question 3 by indicating whether it is true or false.

1. A learner can determine the exact number of objects in a group by ……………………..….
2. When we tell the position of an object in relation to other objects to a learner, we are using number in the ………………… sense.

3. In counting, the order in which the learner says the number names does not matter. True or false
4. Learners have difficulty understanding the number zero. Describe briefly how you would introduce it to pupils in Primary Class one.

5. Explain briefly what a cardinal number is to a Primary Class Two pupil.

SESSION 2: DEFINING BASIC ARITHMETIC OPERATIONS (ADDITION, AND SUBTRACTION, MULTIPLICATION, AND DIVISION AND THEIR PROPERTIES)

In this session we shall discuss how we will introduce addition, subtraction, multiplication, and division of whole numbers to learners. Here, we will learn how to help learners build up early number facts involving these operations. To do this we will create suitable activities using mainly concrete materials in order that learners can discover the basic number facts for themselves. They will eventually be required to commit these to memory. Later, when the learners are quite conversant with the use of real or physical objects to discover number facts, they can be guided to use also pictures of objects put in groups and the number line to find these facts.

Session learning outcome(s)

By the end of the session, you should be able to guide learners to:

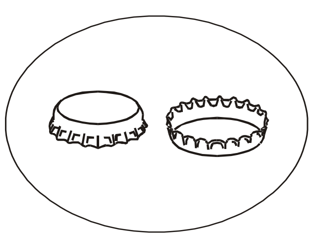
1. add two 1-digit whole numbers
2. subtract two 1-digit whole numbers
3. multiply two 1-digit whole numbers
4. divide two 1-digit whole numbers

2.1 Adding two 1-digit whole numbers (Meaning of addition)

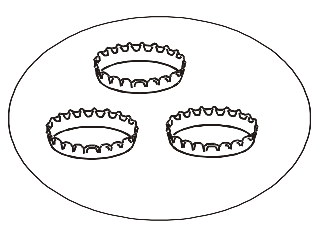
We introduce addition as an operation involving numbers. We can use basically three ways to introduce the addition operation. The first involves the use of groups of objects, while the second and third approaches involve the use of Cuisenaire rods and number line respectively.

(a) Using groups of objects

In this approach, we ask learners to form two groups of objects one with, say, two objects and the other three (Fig 2.4).



2 bottle tops



3 bottle tops

Fig 2.4

We then ask them to put the objects in the two groups together and count to determine the number of objects in the new group formed (Fig 2.5).

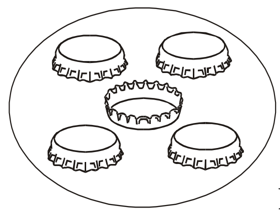


Fig 2.5

The simple language that goes with this activity will be “two bottle tops put together with three bottle tops give five bottle tops”. Later we introduce the formal language and notation of addition to the children.

We lead them to say, “Two plus three equals five”, and write 2 + 3 = 5”.

(b) Using Cuisenaire Rods

We can teach addition of whole numbers using Cuisenaire rods. The rods have different colours and lengths and represent different numbers from 1 to 10.

To add 2 and 3 we take a red rod which represents 2 and a light green rod which represents 3 and place them end to end. The learners then find another rod to match the total length. They discover that a yellow rod which represents 5 matches the total length of the two rods (Fig. 2.6).

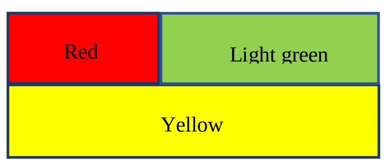


Fig 2.6

Thus, the learners will notice that 2 + 3 = 5 read “two plus three equals five”.

(c) Using Number Line

We can guide learners to use the number line to add numbers. They draw straight lines on the floor and mark on them points with equal spaces between. They assign whole numbers to these points starting with 0 at the left end.

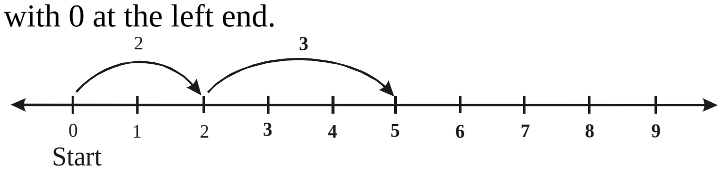


Fig. 2.7

To add 2 and 3, learners start at 0, jump two spaces to 2, jump 3 spaces from 2 and end on 5. (Fig. 2.7). Thus, the learners notice that 2 + 3 = 5. These approaches will involve learners in the initial understanding of addition. We need to guide the learners to build up the basic addition facts 0 + 1=1, 0 + 2 = 2 and so on up to 9 + 9 = 18. They should be able to record these accurately and to remember them for all time. They need **not** commit to memory number facts beyond 9 + 9 = 18 as there are other methods (known as algorithms) that they can use to arrive at answers.

Furthermore, we should discuss with the learners the commutative and associative properties of addition, as well as the identity element for addition. The discussion of these properties is important as learners can use them, when well understood, to simplify their addition problems.

(d) Commutative , associative and identity properties of addition.

Commutative property of addition

We introduce to learners the fact that addition can be performed in both ways to give the same answer. For example, 2 + 3 = 3 + 2. With primary learners, we need not use the word ‘commutative’. The idea can be learned as they discover the number facts in both ways: 2 + 3 = 5 and 3 + 2 = 5. Let us now discuss two ways by which learners can discover the commutative property of addition using concrete approach.

i. Use of Cuisenaire rods

Lead learners to make a train of red rod and light green rod (2.13a).

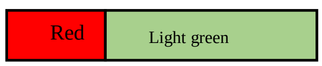


Fig. 2.13a

Next, assist learners also to make a train of light green and red rod (2.13b).

Fig. 2.13b

Assist learners to compare the lengths of the two set of train of rods by placing them side by side as shown Fig. 2.13c.

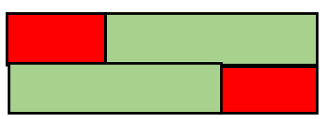


Fig. 2.13c

Lead learners to discover that they are of the same length and also to conclude that associating the rods with their number names and therefore numerals:

$$2 + 3 = 3 + 2$$

Repeat this activity using other train of Cuisenaire rods.

ii. Use of number line.

Similarly the number line can also be used to show the commutative property of addition as illustrated in Fig 2.14

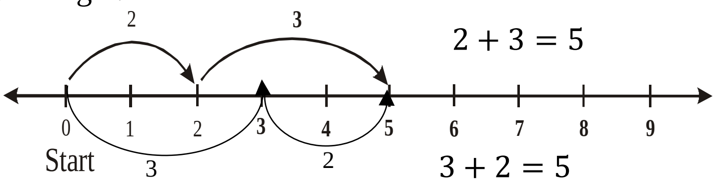


Fig. 2.14

Conclusion: $2 + 3 = 3 + 2$

Associative property of addition

We need to introduce learners to this property when they add three numbers. Since addition is an operation on two numbers, we guide the learners to add two of the three numbers first. For example, to find the sum of 2, 3 and 4, we guide them to add first 2 and 3, and then add the third number 4 to the result.

That is,

$(2 + 3) + 4 = 5 + 4 = 9$.

We can also ask them to add the last two of the three numbers first. That is,

$2 + (3 + 4) = 2 + 7 = 9$.

This means that the way we group the numbers in addition problem does not affect the answer (the sum). In this case also, we need not use the word ‘associative “when we are helping learners in primary school to discover this property. The order of adding in addition does not change the sum.

Addition property of zero (Identity property)

By considering a group of objects put together with another which has no objects in it, we can lead learners to discover

Basic addition facts such as:

$1 + 0 = 1$ and $0 + 1 = 1$
$2 + 0 = 2$ and $0 + 2 = 2$
$5 + 0 = 5$ and $0 + 5 = 5$

Learners can infer from these that when zero is added to a number the result is that same number to which zero is added.

2.2 Subtracting two 1- digit whole numbers (Meaning of subtraction)

We consider subtraction under three approaches: take away, comparison, and finding missing addend. In the subtraction sentence $5 - 2 = 3$, 5 is called the minuend, 2 the subtrahend, and 3 the difference.

(a) Take away

For example, for the subtraction problem 5 – 2, we read this as 5 take away 2 and model this as having five objects (pebbles) and removing 2 objects (pebbles) from it.

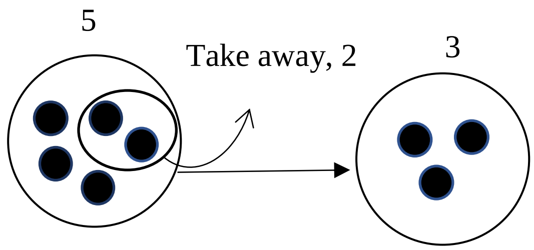


Fig 2.15

We say that when we take away two pebbles from a group of five pebbles we have three pebbles left (Fig. 2.15).
We thus conclude that 5 take away 2 gives 3. That is $5 - 2 = 3$

We can also use the number line approach to explain the idea of subtraction as take away. An example is shown in Fig. 2.16

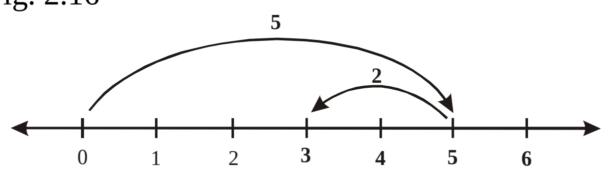


Fig. 2.16

We thus conclude that 5 take away 2 gives 3. That is $5 - 2 = 3$

(b) Comparison

Considering the same subtraction problem, 5 – 2, we lead learners to model two groups of objects (pencils) containing 5 pencils and 2 pencils respectively. We then assist learners to compare the two groups by matching the objects (pencils) on one-to-one basis (Fig. 2.17). Learners are led to conclude that the number of objects (pencils) unmatched (3) is the answer to the subtraction problem. That is $5 - 2 = 3$

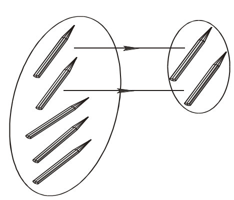


Fig. 2.17

(c) Finding the missing addend

Subtraction can also be interpreted as finding the missing addend. Here, learners are giving for example, the addition sentence $2 + \triangle = 5$, to find the missing number, the addend. With the help of Cuisenaire rods or the activity of counting on, learners are then assisted to find the missing addend as 3. With more examples, learners a led to discover that answer to addition sentence can be obtained by subtracting the addend 3 from the sum 5. Thus, answer to $2 + \triangle = 5$ can be obtained by finding $\triangle = 5 - 2$.

2.3 Multiplying two 1-digit whole numbers (Meaning of multiplication)

(a) Developing the concept of multiplication and ways of doing multiplication

Historically, multiplication was developed to replace certain special cases of addition, namely, addition of *equal addends*. For this reason, we usually see multiplication of whole numbers as **repeated addition.** [Note that Multiplication is introduced in Primary Class 2]
To teach the concept of multiplication to learners, it would be advisable to start with multiplication by 2. Use real objects like counters (counting sticks, bottle tops) and group them in pairs.

For example, in Fig.2.18, objects (e.g., bottle tops) have been grouped in pairs.

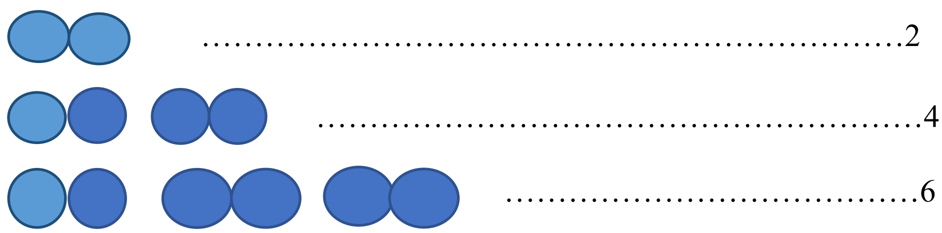


Fig. 2.18

The first group is a pair of bottle tops – 2 bottle tops. The second group is 2 pairs of bottle tops – 4 bottle tops and so on. How many bottle tops will you have all together in 6 pairs of bottle tops? The set of numbers 2, 4, 6, 8, 10………. gives the result of the multiplication table for 2. This result can also be obtained by means of ***skip counting.*** The grouping can also be done with three counters, four counters, and so on.

This process of development leads learners to the discovery of all the multiplication facts associated with two, three or four – i.e., the multiplication tables for two, three or four. Multiplication may also be considered as repeated addition. For example, 3 groups of 2 or 3 twos = 2 + 2 + 2 = 6.
The number line can also be used to find product of two whole numbers. For example, in Fig.2.19, the number line shows 3 groups of 4 or 3 fours. The result is 12.

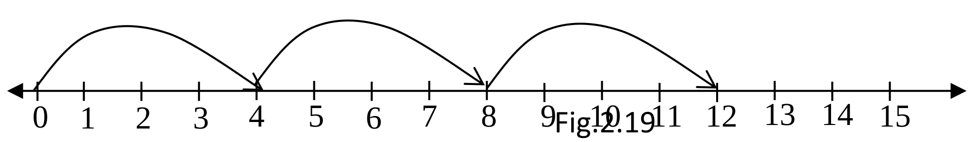


Fig.2.19

The figure above shows 3 groups of 4 or 3 fours and it is written as:

$$3 \times 4 = 12.$$

The sentence 3 groups of four (or 3 fours) is the same as 4 groups of three (or 4 threes).
That is 

This is shown in Fig.1.23 below:

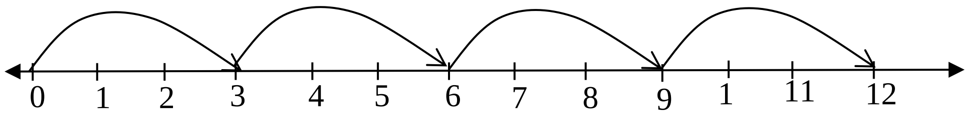
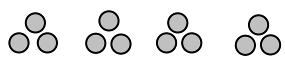


Fig. 2.20

Now, 4 groups of three (or 4 threes) = $4 \times 3 = 12$. This shows that multiplication of whole numbers is commutative. In $4 \times 3 = 12$, 4 is called the *multiplicand*, 3, the *multiplier* and 12, the *product.* Learners can be helped to build multiplication table by using concrete objects like match boxes. They can also build multiplication chart as follows:

x	1	2	3	4	5	
1	1	2	3	4	5	
2	2	4	6	8	10	
3	3	6	9	12	15	
4	4	8	12	16	20	
5	5	10	15	20	25	etc.

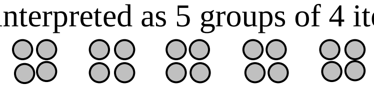
We have seen that 4 threes means 4 groups of 3 items each. In other words, 4 threes mean the same as:



$4 \times 3 = 3 + 3 + 3 + 3 = 12,$
$4 \times 3 = 12.$

Thus, in teaching multiplication of whole numbers, we can use the idea of repeated addition using real objects or counters.

For example, 5×4 is the same as 4 being added to itself five times.
Thus, $5 \times 4 = 4 + 4 + 4 + 4 + 4 = 20.$

This can also be interpreted as 5 groups of 4 items. That is:


Counting all the items gives 20 items. Therefore $5 \times 4 = 20$.
The number line may also be used in teaching multiplication of whole numbers.
For example, 4×6 = 4 groups of 6 items. This is shown in Fig. 2.21

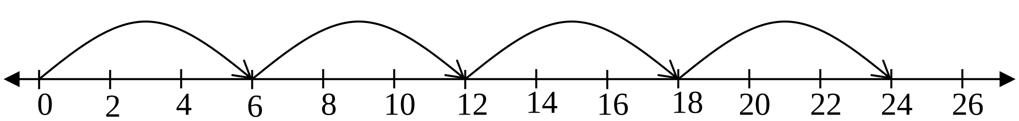


Fig. 2.21

From the number line, 4 groups of 6 items equal 24 items.
Therefore $4 \times 6 = 24$.

Another way of representing multiplication of whole numbers is by use of a **rectangular array.** For example, Fig.2.22 below shows the close relationship between the use of *repeated addition* and *rectangular arrays* for illustrating products. Fig. 2.22*a* shows squares in 5 groups of 7 to illustrate 7 + 7 + 7 + 7 + 7 and Fig. 2.22*b* shows the squares pushed together to form a 5×7 rectangle.

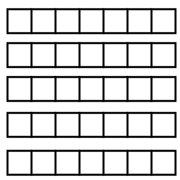


Fig. 2.22*a*

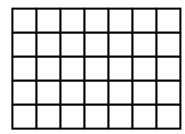


Fig. 2.22*b*

We can also introduce our learners to the ladder approach of finding the product of two whole numbers. Let us consider an example. Our wish is to use the ladder approach to find the product of 2 and 5 (2×5).

With the help of straws or counting sticks, lead the learners to make a ladder as shown in Fig.2.23

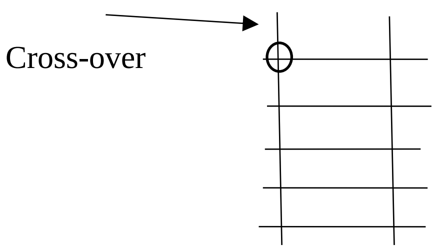


Fig. 2.23

Assist learners to count and record the total number of cross-overs (expected answer is 10).
Lead learners to conclude that the product of 2 and 5 is 10 ($2 \times 5 = 10$).

The Cuisenaire rods can also be used to model multiplication sentences. For example, the multiplication $2 \times 5 =?$ can be modeled by joining two yellow rods end to end Fig.2.24. Learners are led to discover that the result is an orange rod. Thus, $2 \times 5 = 10$

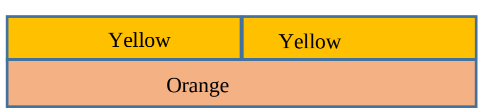


Fig. 2.24

3.3 Dividing two 1-digit whole numbers or a 2-digit whole number by a 1 digit whole number (meaning of division)

In sub-sessions 2.1 and 2.3 we considered *addition* and *subtraction operations on* whole numbers. In this sub-session, we are going to consider another operation on whole numbers called *division*.

We shall attempt to explain the concepts of division and apply these concepts meaningfully, find quotient **of** two given whole numbers, as well as division of a whole number by one and zero.

(a) Division as sharing

The concept of division is best understood if we consider division as *sharing.* For example, let 2 learners share 4 bananas by giving the bananas out to the learners, one at a time. (Suppose the 2 learners are Kojo and Esi.). Then,

Step 1. Kojo takes 1 and Esi takes 1: 2 bananas shared; 2 bananas left.

Step 2. Kojo takes 1 and Esi takes 1: 4 bananas shared; No banana left

So Kojo gets 2 and Esi gets 2. Thus, 4 bananas is shared equally by the two.

Sharing 4 objects equally between 2 learners is written as 4 ÷ 2 (read as 4 divided by 2) and the result (called quotient) is 2. Let learners do a lot of practical exercises on sharing.

Thus, to teach say 8 ÷ 2, one way is to use the sharing concept of division, where 48 counters are shared equally among 4 learners by giving them out one at a time till all the counters are shared out. Each child can then be asked to tell how many counters he or she receives. It will be found that each child will receive 4 counters (Fig. 2.25).

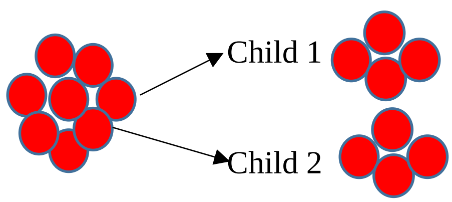


Fig. 2.25

Example, let learners do the following exercises:

Share 6 rulers among 3 boys: 6 ÷ 3 = ………………

Share 3 copybooks among 3 girls: 3 ÷ 3 =………………

Share 9 sticks among 3 men: 9 ÷ 3 = ………………

Share 22 pencils between 2 boys: 22 ÷ 2 =………………

(b) Division as grouping

We can also consider division as *grouping.* Here, we separate a group of objects into smaller groups of equal size.

Example

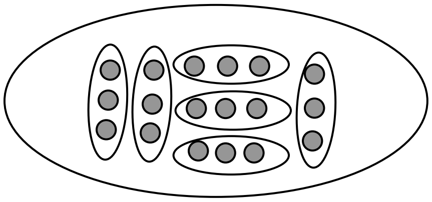


Fig. 2.26

The 18 counters have been separated into smaller groups, each of 3 counters. Counting the smaller groups, we have 6 equal sub-groups. This can be written as 18 ÷ 3 = 6 (Fig. 2.26).

This division fact can also be demonstrated on the number line.

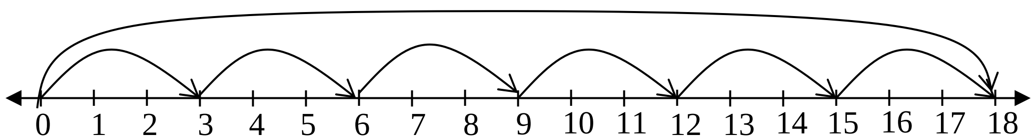


Fig. 2.27

In Fig.2.27, the number line shows that there are 6 groups of 3 in 18. In other words, Fig.2.27 illustrates $6 \times 3 = 18$ or $18 \div 3 = 6$.

Thus, to teach $48 \div 4$ using the concept of division as grouping, one has to form as many groups of 4 counters as possible from 48 counters. In this case there will be 12 groups of 4 counters each, as shown below. The *number* of groups, namely 12, is the quotient of $48 \div 4$ (Fig 2.28).

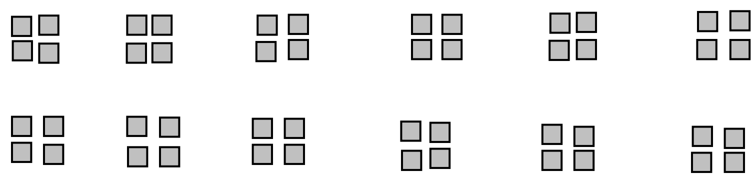


Fig. 2.28

(c) Division as finding the missing factor

We can also consider division as *'finding the missing factor'*.

For example: $2 \times \square = 8$. Finding the missing factor is the same as sharing (dividing) 8 items among 2 i.e. $2 \times \square = 8 \Rightarrow = 8 \div 2$. Therefore, $\square = 4$, because $2 \times 4 = 8$.

This approach of interpreting division can also be referred to as treating division as the inverse operation of multiplication.

For example, $8 \div 2$ can be interpreted as $\square \times 2 = 8$ (what number multiplied by 2 gives 8?)

By using multiplication facts, learners will discover the missing factor to be 4. Thus, $8 \div 2 = 4$.

(d) Division as comparing (ratio)

Another common use of division is to compare two quantities. Consider the two containers below. Container A holds 3 litres of liquid and B holds 45 litres of liquid. We can determine number of container A required to fill container B. This can be done by dividing 45 by 3 (Fig.2.29). That is $45 \div 3$ and the answer is 15.

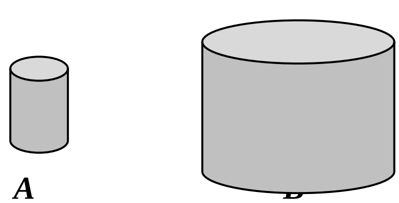


Fig.2.29

In other words, 45 ÷ 3 = 15 because 15 × 3 = 45. This explains why *multiplication* and *division* are called ***inverse operations.***

Furthermore, 48 ÷ 4 can be found by considering division as the missing factor in a multiplication sentence. Hence to find 48 ÷ 4, we rather find the value of the '*box*' such that

$4 \times \boxed{} = 48$. Since 4 × 12 = 48, then 48 ÷ 4 = 12.

(e) Terms used in division process

There are 3 basic terms used in describing the division process, namely *dividend, divisor,* and *quotient*. In 45 ÷3 = 15, 45 is the dividend, 3 is the divisor and 15, is the quotient. For any whole numbers r and s, with $s \neq 0$, the quotient of r divided by s, written $r \div s$, is the whole number k, if it exists, such that $r = s \times k$.

Key Ideas

- Addition means putting together two or more groups
- Teaching addition can be done using:
  (a) groups of objects
  (b) Cuisenaire rods
  (c) The number line
- The numbers in an addition sentence are called addends and the answer is called the sum
- Change in the order of the addends in an addition sentence gives the same sum.
- The order of adding in addition does not change the sum
- When zero is added to a number the result is that same number
- Subtraction means take away, comparison and finding the missing addend.
- The three numbers in a subtraction sentence are respectively called minuend, subtrahend and difference.
- Multiplication is repeated addition.
- The numbers a multiplication sentence *are* called *multiplicand, multiplier* and *product respectively*. However, the *multiplicand* and *the multiplier are* also called factors.
- Division is repeated subtraction, grouping and finding the missing factor.
- The three numbers involve in a division sentence are called dividend, divisor and quotient respectively.
- Addition is the inverse of subtraction and vice versa.
- Multiplication is the inverse of division and vice versa.

Self- Assessment Questions

Answer questions 1 and 2 by filling the blank spaces and question 3 by indicating whether it is true or false.

1. A learner can determine the exact number of objects in a group by ……………………..…

2. When we tell the position of an object in relation to other objects to a learner, we are using number in the ………………… sense.
3. In counting, the order in which the learner says the number names does not matter. True or false
4. Learners have difficulty understanding the number zero. Describe briefly how you would introduce it to pupils in Primary Class one.
5. Explain briefly what a cardinal number is to a Primary Class Two pupil.
6. Describe two different approaches you would use to help learners in Basic Class I to solve the problem 6 – 2.
7. Describe clearly how you will use concrete materials to help learners in Basic 3 to solve the division problem 8 ÷4 using the method of:
   (a) Sharing (b) Grouping
8. Explain how you would help a learner in Basic 6 to solve the following problems using teaching-learning materials (a) $4 \times \frac{3}{5}$ (b) $3 \div \frac{1}{2}$

SESSION 3: DEVELOPING THE CONCEPT OF COMMON FRACTIONS AND PERFORMING BASIC OPERATIONS ON COMMON FRACTIONS

The language of fraction is used in our everyday conversation to simply mean part of a whole, a set or a measured quantity. The concept of fractions can be studied in three different areas: common fractions, decimal fractions, and percentage fractions. We shall in this session focus our deliberations on the concept of common fractions.

Session learning outcomes

By the end of this session, you should be able to guide learners to:

1. explain the concept of common fraction.
2. describe common fractions as part of a whole, as part of a group, as a quotient of two whole numbers.
3. discover different types of common fractions.
4. compare and order common fractions
5. perform basic operations on common fractions

3.1 Explaining the concept of common fraction

The term *fraction* is used to refer to a number written in the form $\frac{a}{b}$. The *top* number, *a*, is called ***numerator*** and the *bottom* number, *b*, is called ***denominator.*** Thus in $\frac{4}{7}$, 4 is the *numerator* and 7, the *denominator*. Common fractions comprises, proper, improper, and mixed fractions. Common fractions are made up of proper, improper, and mixed numbers (fractions). Examples are: $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{12}{15}$. Improper fractions have their numerators greater than their denominators.

Examples are: $\frac{5}{2}$, $\frac{8}{3}$, and $\frac{13}{12}$. Mixed numbers (fractions) are however a combination of natural numbers and proper fractions. Examples are $1\frac{1}{5}$, $3\frac{3}{8}$, and $10\frac{11}{13}$. Common fractions are usually introduced to learners as part of a whole or part of a group or quotient of two whole numbers. We shall further discuss these two in detail.

3.2 Describing common fractions: as part of a whole, as part of a group, and as a quotient of two whole numbers

(a) Common Fraction as Part of a Whole (One-half and One fourth)

Put your learners into groups of twos with each group having a sizeable sheet of different shaped (rectangular, square, and circular) papers to fold. Let each group fold the sheet of paper into two equal parts. Let each group shade one part of two equal parts with a colour of their choice (Fig.2.37). Assist each group to identify each equal part with the number name, *one-half*. Let groups with the same shapes compare the different ways they did their folding. Next, introduce the numeral, $\frac{1}{2}$ (read as 'one-out of two') as the symbolic representation of the number name, one-half (Fig.2.30).

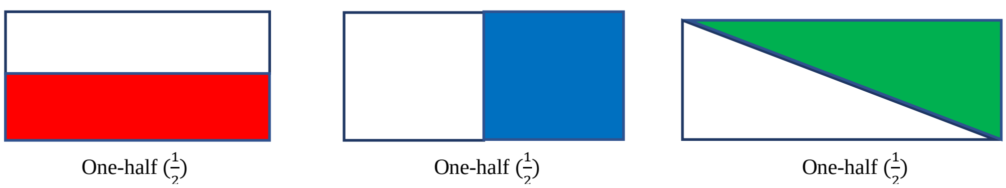


Fig.2.30

Next, give each group another set of papers to fold into four equal parts. Let each group shade one part of the four equal parts with a colour of their choice. Assist each group to identify each folded part with the number name, *one-fourth or one-quarte*r (Fig.2.31). Let groups with the same shapes compare the different ways they did their folding. Next, introduce the numeral, $\frac{1}{4}$ (read as 'one-out of four') as the symbolic representation of the number name, one-fourth.

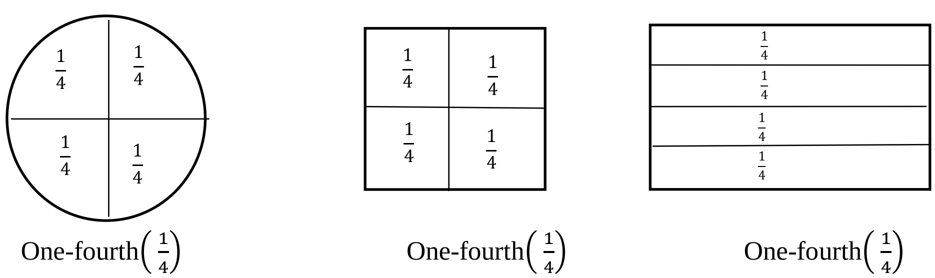
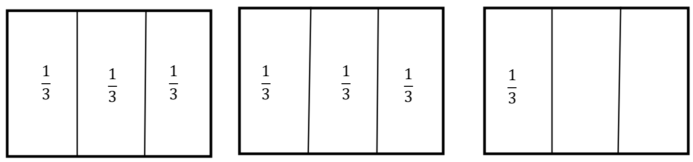


Fig.2.31

Repeat this activity for one-eighth $\left(\frac{1}{8}\right)$ and possibly, for one-third $\left(\frac{1}{3}\right)$.

Learners may now be led to list the multiples of each of the common fractions learnt. For example, the multiples of one-half are one-half $\left(\frac{1}{2}\right)$, two-halves $\left(\frac{2}{2}\right)$, three-halves $\left(\frac{3}{2}\right)$ etc. Also, the multiples of one-fourth (one-quarter) are one-fourth $\left(\frac{1}{4}\right)$, two-fourths $\left(\frac{2}{4}\right)$, three-fourths $\left(\frac{3}{4}\right)$, four-fourths $\left(\frac{4}{4}\right)$, five-fourths $\left(\frac{5}{4}\right)$etc. Similarly, you may assist your learners to list the multiples of one-third $\left(\frac{1}{3}\right)$ and one-eighth $\left(\frac{1}{8}\right)$ in like manner.

A learner after this activity should be able to model or make for example, seven-thirds $\left(\frac{7}{3}\right)$, as shown in Fig.2.32.



(b) Common fractions as part of a group

Fig. 2.22a

At times fractions may be considered as part of a group. For example, in a group of *ten*, if we are interested in one-half $\left(\frac{1}{2}\right)$, then we must separate the group into two separate equal sub-groups. Thus, each sub-group may be considered as one-half. Diagrammatically this may look like the one in (Fig.2.33). Mathematically, each sub-group is written as $\frac{1}{2}$.

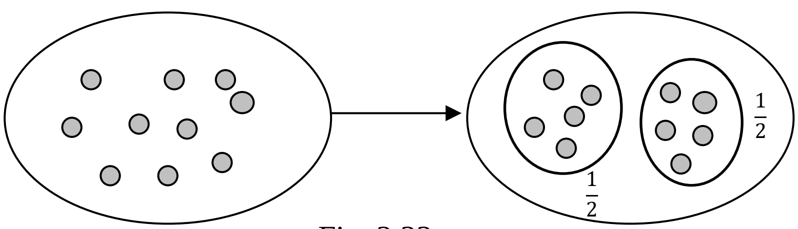


Fig. 2.33

Similarly, if learners separate a group of 20 objects (bottle tops) into 4 equal sub-groups (Fig.2.34), then each sub-group will represent one-quarter, $\left(\frac{1}{4}\right)$.

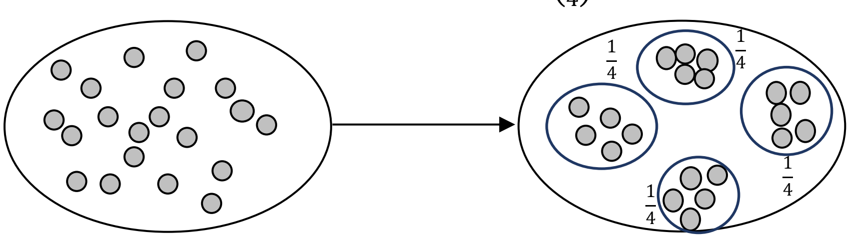


Fig.2.34

(c) Common fractions as comparing (division) of two quantities of the same measure (Ratio)

We have already considered *division of whole numbers*. At times a fraction is considered by means of *division concept.* For example, dividing 25 by 50 means there will be 50 *equal* parts and taking 25 parts. Remember, this is written as $25 \div 50 = \frac{25}{50}$.

Again, another use of fractions involves the *ratio concept.* In this case, fractions are used to compare one amount to another. For example, we might say that a girl's age is *one-third* her mother's age.

The ratio concept of fractions can be illustrated with the *Cuisenaire Rods* by comparing the lengths of two rods.

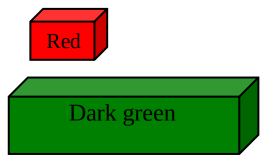


Fig.2. 35

For example, in Fig.2.35, it takes 3 red rods to equal the length of 1 dark green rod. If the dark green rod is chosen as the unit, then the red rod represents $\frac{1}{3}$.

Similarly, it takes 2 yellow rods to equal the length of 1 orange rod (investigate). If the orange rod is taken as a unit, then the yellow rod represents $\frac{1}{2}$.

3.3 Discovering different types of common fractions.

Common fractions are of different types. Write for example on the board, the following common fractions:

$$\frac{2}{3}, \quad \frac{5}{3}, \quad 1\frac{2}{3}, \quad \frac{7}{5}, \quad \frac{3}{4}, \quad 3\frac{1}{2}, \quad \frac{9}{13}, \quad \frac{15}{11}, \quad \frac{7}{2}, \quad 5\frac{1}{8}$$

Assist learners to sort the fractions into three distinct groups, such as:

A. $\frac{2}{3}, \frac{3}{4}, \frac{9}{13}$

B. $\frac{5}{3}, \frac{7}{5}, \frac{15}{11}, \frac{7}{2}$

C. $1\frac{2}{3}, 3\frac{1}{2}, 5\frac{1}{8}$

Lead learners to:

a) Identify group **A** as proper common fraction (fractions with numerator less than the denominator).

b) Identify group **B** as improper common fractions (fractions with numerator greater than the denominator).

c) Identify group **C** as mixed numbers (fractions) (fraction with a whole number and a proper common fraction).

You may assist your learners to realise that some of the proper and improper common fractions have the same denominator (Like common fractions) and others have different denominators (unlike common fractions).

3.4 Comparing and ordering common fractions

This sub-session will be treated in two parts: comparing like –common fractions and comparing unlike common fractions.

(a) Comparing like-common fractions

To compare like-fractions such as $\frac{3}{4}, \frac{1}{4}, \frac{5}{4}\, and\, \frac{2}{4}$, we compare the numerators to identify the greatest. In the given fractions, 5 is greater than 3; 3 is also greater than 2; and finally, 2 is greater than 1. Hence, we can write the relations between the above fractions as:

$$\frac{5}{4} > \frac{3}{4} > \frac{2}{4} > \frac{1}{4}.$$

This can be illustrated by using *paper folding* or the *number line.* Illustration of these fractions on the number line is shown in Fig.2.36

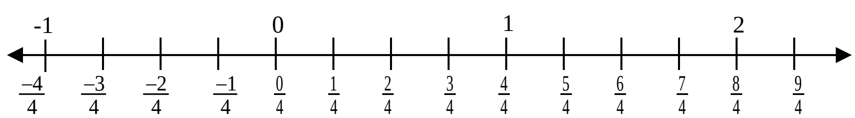


Fig.2.36

To compare unlike-common fractions, we need to learn *equivalent fractions.*

Equivalent fractions: Equivalent fractions can be illustrated by comparing parts of a whole as illustrated in Fig 2.37.

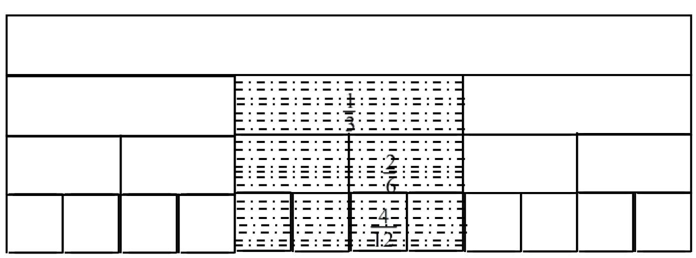


Fig 2.37

Fig.2.44 shows that $\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = ...$

Write the next four equivalent fractions.

Fundamental rule for writing equivalent fractions is that:

$$\textit{for any fraction } \frac{a}{b} \text{ and any number } k \neq 0,\ \frac{a}{b} = \frac{ka}{kb}.$$

For example $(a)\ \frac{1}{2} = \frac{1\times 7}{2\times 7} = \frac{7}{14}$ $\quad (b)\ \frac{3}{4} = \frac{3\times 2}{4\times 2} = \frac{6}{8}$ $\quad (c)\ \frac{2}{3} = \frac{2\times 5}{3\times 5} = \frac{10}{15}$

To compare the fractions $\frac{2}{5}$ and $\frac{1}{3}$, we write the equivalent fractions of these fractions till we arrive at the two with the same common denominator (i.e., like-fractions for the two fractions).

For example, $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \ldots\ldots$ and $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18} = \ldots\ldots$

The like-fractions for the two fractions are $\frac{6}{15}$ and $\frac{5}{15}$.

Hence $\frac{2}{5} = \frac{6}{15}$ and $\frac{1}{3} = \frac{5}{15}$. Since $5 < 6$, we conclude that $\frac{2}{5} > \frac{1}{3}$.

3.5 Basic operations on common fractions

In 3.1 we considered fraction as a concept and we saw that a fraction may be considered as *a part of a whole*, *a part of a group* or as a *division*. In this session, we shall consider operations on common fractions. In other words, we shall look at addition, subtraction, multiplication, and division of common fractions. The discussion will be done as follows:

(a) addition and subtraction like common fractions (common fractions with the same denominator);
(b) multiplication of a whole number and common fractions or multiply a common fraction and a whole number;

(c) division of a whole number by a common fraction or divide a common fraction by a whole number.

(a) Addition and subtraction of like Common Fractions

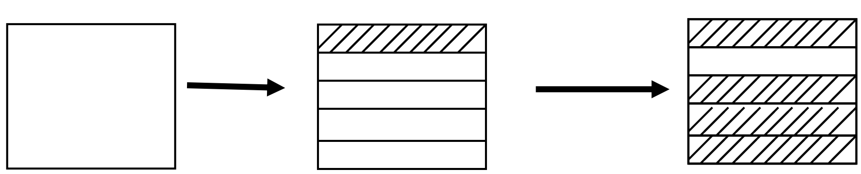
(i) Addition of like common fractions

Using paper folding

It has already been explained that like fractions are common fractions with the same denominator.

Examples are: $\frac{2}{5}$ and $\frac{4}{5}$; $\frac{5}{8}$ and $\frac{3}{8}$; and so on.

(a) To add for example, $\frac{1}{5}$ and $\frac{3}{5}$, we can perform this operation by means of the following diagrams:



$\frac{1}{5}$

$\frac{3}{5}$

Fig. 2.38

$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$ (By counting the shaded unit rectangles).

The steps involve are as follows:

- Folding a sheet of rectangular piece of paper into 5 equal parts (equal unit rectangles).
- Shading 1-unit rectangle to represent the fraction $\frac{1}{5}$.
- Shading 3-unit rectangles, representing $\frac{3}{5}$, in addition.
- Counting the unit rectangles shaded altogether (4-unit rectangles shaded altogether).
- Representing the 4-unit rectangles shaded altogether by $\frac{4}{5}$.
- Concluding that $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$

One shaded unit rectangle and three shaded unit rectangles give <u>four</u> shaded unit rectangles out of five-unit rectangles.

Again, if we were to find $\frac{2}{5} + \frac{4}{5}$ by the same way, then we have to take learners through the following steps:

- Get learners to fold or divide a rectangular or square sheet of paper into five equal unit rectangles or squares
- Let learners shade two unit squares to represent $\frac{2}{5}$.
- Assist learners to shade additional four unit squares, representing $\frac{4}{5}$ on the same sheet of paper.
- After shading the three un-shaded remaining unit rectangles or squares, learners will realise that one unit rectangle or square will be left to be shaded.
- Let learners pick a congruent rectangular or square sheet of paper, fold or divide it into five equal unit rectangles or squares, the same way as the first and shade the remaining one unit rectangle or square on it (see Fig. 2.39).
- Finally, assist learners to conclude that $\frac{2}{5} + \frac{4}{5} = \frac{6}{5}$ or $1\frac{1}{5}$.

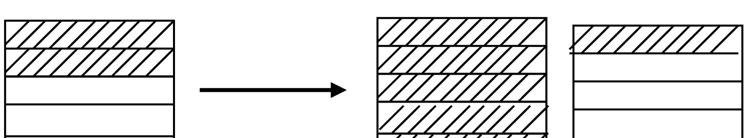


Fig. 2.39

$$\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\tfrac{1}{5}$$

Note that this is an example of *improper fraction.* It can be rewritten as *mixed number (fraction).* Thus $\frac{6}{5} = 1\frac{1}{5}$

Using number line

The addition of like common fractions can also be performed by means of the *number line.*

We show $\frac{1}{5} + \frac{3}{5}$ on the number line as follows (Fig.2.40).

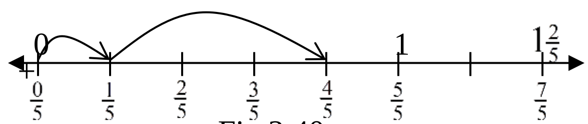


Fig.2.40

When adding like fractions like $\frac{1}{5}$ and $\frac{3}{5}$, because the two fractions have the same denominator 5, we can write $\frac{1}{5} + \frac{3}{5} = \frac{1+3}{5} = \frac{4}{5}$.

The common denominator (here it is 5) is called the *least common multiple* or simply the LCM of the two fractions. Thus, to find the sum of $\frac{3}{8}$ and $\frac{7}{8}$, we write

$$\frac{3}{8} + \frac{7}{8} = \frac{3+7}{8} = \frac{10}{8}$$

Now, $\frac{10}{8}$ is an improper fraction, which can rewrite it as $1\frac{1}{4}$ (a mixed number).

Thus, $\frac{3}{8} + \frac{7}{8} = \frac{10}{8} = 1\frac{2}{8} = 1\frac{1}{4}$

(ii) Subtraction of like common fractions

The subtraction of common fractions is done in similar way as subtraction of whole numbers. That is, the *take-away concept* and the *missing-addends concept* also apply to the subtraction of common fractions. For subtraction of like common fractions, example, $\frac{5}{7} - \frac{2}{7}$:

(i) We can use the number line as illustrated in Fig.2.41 below.

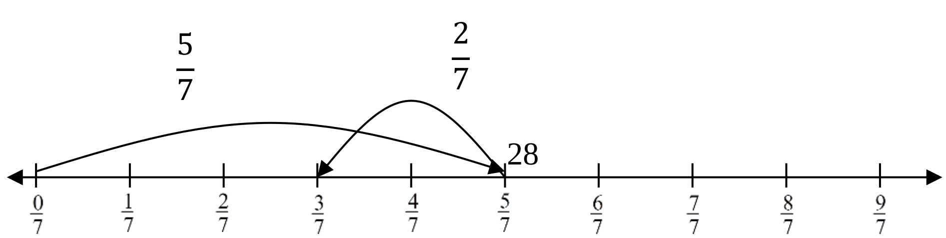


Fig.2.41

We start from zero and count five units (out of seven) to the right i.e., $\frac{5}{7}$. We then count backwards two units (out of seven) to the left i.e.,$\frac{2}{7}$

. Thus, we end on $\frac{3}{7}$

Therefore, we conclude that $\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$.

(ii) We can also use bars to show this. This is illustrated in Fig.2.42.

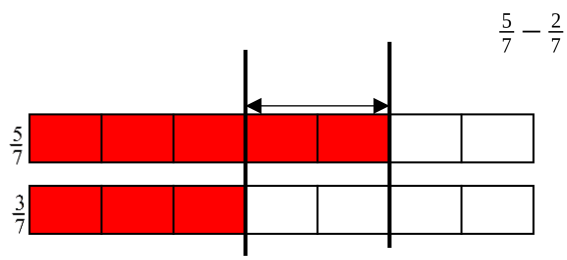


Fig.2.42

The top bar shows $\frac{5}{7}$ part of the whole shaded and the down shows $\frac{2}{7}$ part of the $\frac{5}{7}$ un-shaded. If we take away $\frac{2}{7}$ from $\frac{5}{7}$, we are then left with $\frac{3}{7}$.

Again, since the fractions have the same denominator (LCM), we can write

$$\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}$$

Learners can also be led to fold a rectangular piece of A4 sheet into 7 equal parts (strips), shade 5 parts to represent, $\frac{5}{7}$, un-shade 2 parts to show the removal of $\frac{2}{7}$, and finally, find the fraction left shaded as $\frac{3}{7}$ (Fig. 2.43). Learners are led to conclude that $\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$.

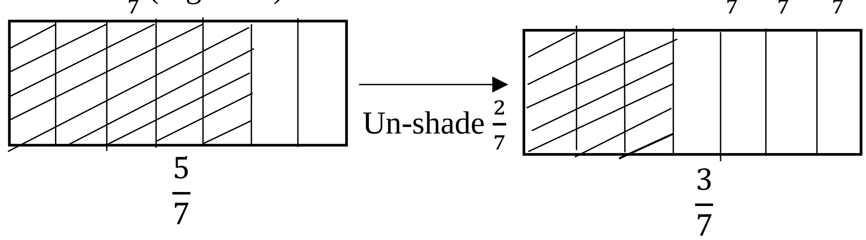


Fig. 2.43

(b) Multiplication of a common fraction by a whole number and vice versa

In Unit 2 session 3, we considered multiplication as repeated addition. For example, 2 × 3 is the same as 2 of 3s or 2 groups of 3 each. This is shown in Fig.2.48 below.

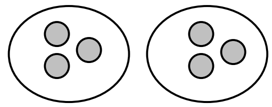


Fig. 2.48

Multiplication of fractions by whole numbers may be viewed in a similar manner.

(i) We may interpret a whole number multiplied by a common fraction to mean *repeated addition of the common fraction.* The whole number indicates the number of times the fraction is to be added to itself. For example, Fig.2.49 illustrates $3 \times \frac{2}{7}$ and shows that this product equals $\frac{6}{7}$

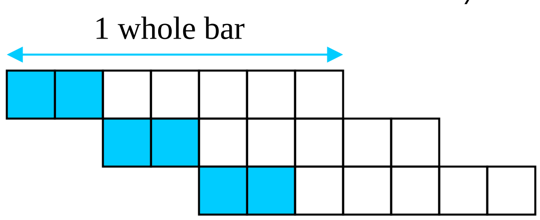


Fig.2.49

Thus, this product may be interpreted as

$$3 \times \frac{2}{7} = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{6}{7}.$$

(ii) A Common fraction multiplied by a whole Number

The product $\frac{1}{3} \times 4$ means $\frac{1}{3}$ of 4. This product can be illustrated by using 4 bars and dividing each into 3 equal parts. Fig.2.50 shows that $\frac{1}{3} \times 4 = \frac{4}{3} = 1\frac{1}{3}$

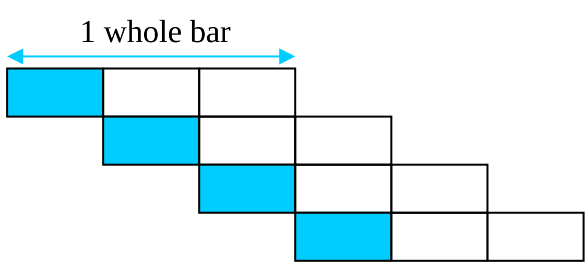


Fig. 2.50

These examples suggest that, for any whole number k and fraction $\frac{a}{b}$

$$k \times \left(\frac{a}{b}\right) = \left(\frac{a}{b}\right) \times k = \frac{ka}{b}.$$

(c) Division of a common fraction by a whole number and vice versa

i. Division of a common fraction by a whole number

Division of fractions may be viewed in much the same way as division of whole numbers. One of the meanings of division of whole numbers is the *sharing concept*.

(i) For example, 15 ÷ 3 may be interpreted as sharing 15 objects among 3 learners. Each learner gets 5. That is 15 ÷ 3 = 5. Hence, $\frac{1}{2} \div 3$ may be interpreted as sharing one-half into three equal parts. This is illustrated in Fig.2.54. It shows that $\frac{1}{2} \div 3 = \frac{1}{6}$

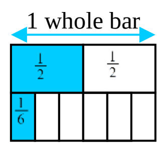


Fig. 2.54

Similarly, $\frac{2}{3} \div 4$ may be interpreted as sharing two-thirds into 4 equal parts. This is also illustrated in Fig.2.55. It again shows that $\frac{2}{3} \div 4 = \frac{1}{6}$.

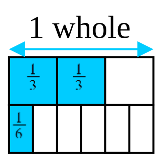


Fig. 2.55

ii. Dividing a whole number by a common fraction

(i) The concept of division as grouping or repeated subtraction could be used to help student divide a whole number by a common fraction. Let us consider for example, $4 \div \frac{1}{2}$. This division sentence can be interpreted as repeated subtraction.

$$4 \div \frac{1}{2} = 4 - \frac{1}{2} = 3\frac{1}{2}$$

$$3\frac{1}{2} - \frac{1`}{2} = 3$$

$$3 - \frac{1`}{2} = 2\frac{1}{2}$$

$$2\frac{1`}{2} - \frac{1}{2} = 2$$

$$2 - \frac{1}{2} = 1\frac{1}{2}$$

$$1\frac{1`}{2} - \frac{1}{2} = 1$$

$$1 - \frac{1}{2} = \frac{1}{2}$$

$$\frac{1`}{2} - \frac{1}{2} = 0$$

Assist learners to notice that $\frac{1}{2}$ was subtracted from 4 to arrive at zero (0), eight times.

So, $4 \div \frac{1}{2} = 8$.

(ii) One of the early methods of dividing a whole number by a fraction was to replace both numbers (written in fractions) by like fractions. When this is done, the quotient can be obtained by disregarding the denominators and dividing the two numerators. For example, $4 \div \frac{2}{3} = \frac{4}{1} \div \frac{2}{3} = \frac{12}{3} \div \frac{2}{3} = 12 \div 2 = 6$

Similarly, $6 \div \frac{3}{4} = \frac{6}{1} \div \frac{3}{4} = \frac{24}{4} \div \frac{3}{4} = 24 \div 3 = 8$

Again, $3 \div \frac{2}{5} = \frac{3}{1} \div \frac{2}{5} = \frac{15}{5} \div \frac{2}{5} = 15 \div 2 = 7\frac{1}{2}$

One method of dividing a whole number by a common fraction is illustrated below. For example:

$$4 \div \tfrac{1}{3} = \frac{4}{\frac{1}{3}} = \frac{4 \times \frac{3}{1}}{\frac{1}{3} \times \frac{3}{1}} = \frac{4 \times 3}{1} = 12$$

It is from this method that some suggest that:

‘*to divide a whole number by a fraction, we invert the divisor and multiply*’

Applying this rule, $6 \div \frac{3}{8} = 6 \times \frac{8}{3} = \frac{48}{3} = 16$.

Key Ideas

- A fraction is simply part of a whole divided into equal parts or part or a group divided into equal subgroups.
- Fractions are either common fractions, decimal fractions or percentage fractions.
- Common fractions are of the form a ratio of a numerator to a denominator.
- Common fractions are either proper, improper, or mixed numbers.
- Proper common fractions have the numerator to be less than denominator.
- Improper common fractions have the numerator to be greater than the denominator.
- Mixed number common fraction is made up of a whole number and a proper common fraction.

- Two common fractions can be added, subtracted, multiplied and divided.

Self-Assessment Questions

1. What name is given to the lower number of a fraction?
2. A fraction may be regarded as a part of …………. or …………
3. Describe one activity you will use to explain to a learner that $\frac{1}{4}$ is 1 part of a whole of 4 equal parts.
4. How will you name ¢5,000.00 out of ¢30,000.00 as a fraction to a learner in Basic 3?
5. Explain like-fractions and unlike-fractions to a learner in Basic 4.
6. What activity will you ask Basic 5 learners to do to compare two unlike common fractions?
7. Describe one activity that can be used to teach Basic 2 learners to find sum of two like-fractions.
8. Mention one activity that can be used to teach subtraction of unlike fractions in Basic 4.
9. We may interpret multiplication of a whole number by a fraction to learners as…………….
10. One of the early methods of dividing by a fraction was to replace both fractions by………….
11. Explain to a Basic 4 learner the difference between a proper and an improper common fraction.

UNIT 3: TEACHING INVESTIGATION WITH NUMBERS

Patterns play a major role in the solution of problems in all areas of life. Psychologists analyse patterns of human behaviour; meteorologists study weather patterns; astronomers seek patterns in the movements of stars and galaxies; and detectives look for patterns among clues. Finding a pattern is such a useful problem-solving strategy in mathematics that some have called it the *art of mathematics.*

Unit learning outcome(s)

By the end of the unit, you will be able to teach learners how to:

1. Find factors and multiples of natural numbers
2. Generate even, odd, prime, composite numbers and prime factorization
3. Find the Highest Common Factor (HCF) and Lowest Common Multiple (LCM) of numbers.

SESSION 1. FINDING THE FACTORS AND MULTIPLES OF NUMBERS

Session learning outcome(s)

By the end of this session you will be able to teach learners to:

1. find factors of natural number
2. find multiples of natural number

1.1 Teaching learners to find factors of natural numbers

You will recall that in a multiplication sentence such as $a \times b = c$, a and b are called factors and c is called the product.

Lead learners to express any natural number from 1 to 20 as the product of any two natural numbers, using examples. Let us consider some examples.

Example 1 Find the factors of 12.

Solution

- Lead learners to express 12 as the product of any two natural numbers:

$$1 \times 12;\ 2 \times 6;\ 3 \times 4;\ 6 \times 2;\ 4 \times 3;\ 12 \times 1$$

- Assist learners to list all the factors of 12 as 1, 2, 3, 4, 6, and 12 itself.
- Make learners aware that the numbers 1, 2, 3, 4, and 6 are proper factors of 12 and 12 is an improper factor of 12.

Example 2. Find all the factors of 18.

Solution

Expressing 18 as the product of any two natural numbers we have:

$$1 \times 18;\ 2 \times 9;\ 3 \times 6;\ 9 \times 2;\ 6 \times 3;\ 18 \times 1$$

- Assist learners to list all the factors of 18 as 1, 2, 3, 6, 9, and 18 itself

- Make learners aware that the numbers 1, 2, 3, 6, and 9 are proper factors of 18 and 18 is an improper factor of 18.
- With an additional example, assist learners to conclude that: A number is a factor of a given number if the number divides the given number exactly, leaving no remainder.

1.2 Teaching learners to find multiples of a given natural number

You will once again recall that in a multiplication sentence if $a \times b = c$, then c is said to be the product of *a* and *b*. If *c* is the product of *a* and *b*, then we say *c* is a multiple of *a* and *c* is also *a* multiple of *b*.

With the help of plastic bottle tops (counting material), lead learners to find the multiples of for example 2, by engaging them in the following activity:

- Give learners enough of the bottle tops to work with.
- Let learners make several groups of bottle tops such that each group contains two bottle tops.
- Next, assist learners to put the bottle tops together consecutively (successively), counting and recording the total number of bottle tops after each put together.
- Assist learners to record their results as 2, 4, 6, 8, 10, 12, 14, 18, and so on.
- Make learners aware that each number in the result is a multiple of 2.

Repeat this activity for multiples of 3 by engaging learners to make several groups of three bottle tops and putting them together successively to produce the multiples of 3 as 3, 6, 9, 12, 15, 18, 21, and so on.

Using several examples, assist learners to discover and conclude that:

- by skipping counts of a given natural number, one can generate the multiples of the number
- a number is a multiple of a given number if the given number divides the multiple number, leaving no remainder.

Key Ideas

- The natural number *M* is a factor of the natural number *N* if *M* divides *N* without a remainder.
- The natural number *M* is a multiple of the natural number *N* if *N* divides *M* without a remainder.

Self-Assessment Questions

1. Describe how you would use 20 counting objects to help a Basic 5 learner to list all the factors of 20.
2. Write down all the factors of 42.
3. The first 10 multiples of the natural number 6 are ______________________________.
4. Using named physical objects, describe a strategy you would use to assist a Basic 6 learner to list the first ten multiples of 6.

SESSION 2: GENERATING EVEN, ODD, PRIME, COMPOSITE NUMBERS AND PRIME FACTORIZATION

Learning outcome(s)

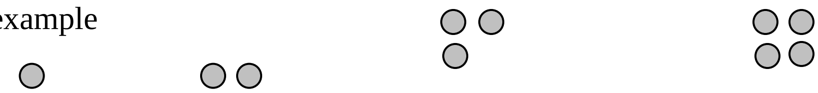
By the end of this session you will be able to teach learners to:

1. Generate even and odd numbers
2. Generate prime and composite numbers
3. Generate the prime factorisation of a natural number

2.1 Generating even and odd numbers

Number patterns have fascinated people since time immemorial. One of the earliest patterns to be recognized led to the distinction between **even** and **odd** numbers. To help learners recognize and write even and odd numbers, let learners use real objects (like counters) and group them in twos.

For example



no group	one group	one group and one	two groups
(not possible)	(possible)	(not possible)	(possible) *etc.*

This could continue up to 10 or even 20 with the lower primary learners. The upper primary learners can continue with higher numbers.

2.2 Generating prime numbers and composite numbers

(a) Finding prime numbers of a natural number

Before introducing learners to *prime numbers*, review with them factors and multiples of whole numbers.

For example:

Factors of 6 are 1, 2, 3, 6 – i.e., 4 factors

Factors of 12 are 1, 2, 3, 4, 6, 12 – i.e., 6 factors, *etc.*

Multiples of 6 are 6, 12, 18, 24, 30, and so on

Multiple of 12 are 12, 24, 36, 48, 60, and so on

Write numbers 1 to 20 on the chalkboard and let learners find the numbers with *only two factors*.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Learners would recognize that the only two factors are **1** and the **number itself.** Let learners extend the number chart to 50 and find all the numbers with only two factors.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Lead learners to name these numbers (numbers with only two factors) as **prime numbers.**

One way of finding all primes that are less than a given number is to eliminate those numbers that are not prime. With the number chart, we begin by crossing out 1, *which is not a prime.* We circle 2 and cross out all remaining multiples of 2 (4, 6, 8, 10, etc.).

Then 3 is circled and all remaining multiples of 3 are crossed out (6, 9, 12, 15....). We note that 4 is not included because, it has more than two factors, i.e., 1, 2 and 4 itself. Again 4 is a multiple of 2. This process is continued by circling 5 and 7 and crossing out their multiples. The numbers that are not crossed out are the **primes.** This method of finding prime numbers less than a given number was first used by a Greek mathematician **Eratosthenes** (230 B.C.). Thus, the method is called **sieve of Eratosthenes.**

Write numbers 1 to 50 on the chalkboard and let learners circle all the numbers that can be grouped in twos.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

The first few numbers that can be grouped in twos have been circled. Let learners sort numbers that can be grouped in twos and name these numbers as **even numbers.** Let learners sort numbers that cannot be grouped in twos and name them as **odd numbers.**

Allow learners to write each even and odd number as the product of any two natural numbers.

Example:

$$6 = 2 \times 3;\ 1 \times 6;\ 3 \times 2;\ 6 \times 1$$

$$12 = 1 \times 12;\ 2 \times 6;\ 3 \times 4;\ 6 \times 2;\ 4 \times 3;\ 12 \times 1$$

$$9 = 3 \times 3;\ 1 \times 9;\ 9 \times 1$$

$$15 = 3 \times 5;\ 5 \times 3;\ 15 \times 1;\ 1 \times 15$$

Assist learners to conclude that:

- an even number has 2 as one of its factors
- even numbers are divisible by 2 (leave no remainder when divided by 2)
- even numbers have their units or ones place value digits as either 0, or 2, or 4, or 6, or 8.
- odd numbers do not have 2 as a factor
- odd numbers are not divisible by 2 (leave a remainder 1 when divided by 2)

(b) Generating composite numbers

Apart from the number 1, let learners find out the factors of all the cross-out numbers in the sieve of Eratosthenes by copying and completing the table shown below:

Cross-out number	Factors	Cross-out number	Factors
6	1, 2, 3, 6		
8	1, 2, 4, 8		
9	1, 3, 8		
10	1, 2, 5, 10		
12	1, 2, 3, 6, 12		
14	1, 2, 7, 14		
15	1, 3, 5, 15		
16	1, 2, 4, 8, 16		

Lead learners to discover that each number **has more than two factors** and help them to identify these numbers **as composite numbers.**

2.3: Finding the prime factorization a natural number

Learners in previous lessons have been introduced to factors and prime factors of whole numbers. A snappy review of these two concepts will facilitate their understanding of the concept of prime factorization. The prime factorization of a natural number is basically about expressing a natural number as the product of its prime factors.

This can be done in two ways:

(a) finding all the prime factors of the natural number and expressing the number as the product of its prime factors
(b) finding prime factorisation of a given natural number by use of the factor tree.

Let us consider each of them using examples.

(a) Finding all the prime factors of the whole number and expressing the number as the product of its prime factors.

Example 1
Write for example, the whole number 12 on the board. Assist learners to list all the factors of 12 as $1, 2, 3, 4, 6, and\ 12$.
Lead learners to list the prime numbers as 2 and 3.
Assist learners to write 12 as the product of 2 and 3 only. That is,

$$12 = 2 \times 2 \times 3.$$

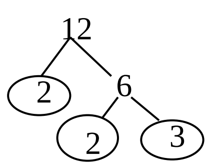
Help learners to conclude that the prime factorization of 12 is $2 \times 2 \times 3$.

Example 2
Write another whole number, for example 18 on the board. Assist learners to list all the factors of 18 as $1, 2, 3, 6, 9, and\ 18$.
Lead learners to list all the prime factors of 18 as 2 and 3.
Assist learners to write 18 as the product of 2 and 3 only. That is $18 = 2 \times 3 \times 3$.
Help learners to conclude that the prime factorization of 18 is $2 \times 3 \times 3$.

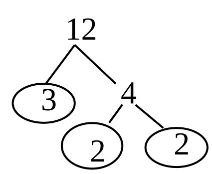
(b) Finding prime factorisation of a given natural number by use of the factor tree
Factor trees are **a way of expressing the factors of a number**, specifically the prime factorization of a number. Each branch in the tree is split into factors. Once the factor at the end of the branch is a prime number, the only two factors are itself and one so the branch stops and we circle the number.

Using example above, you can assist learners to express 12 as the product of its prime factors or simply put, find the prime factorisation of 12, using the factor tree by going through the following steps:

Step 1 Help learners to draw a factor tree for 12 as shown below.



or



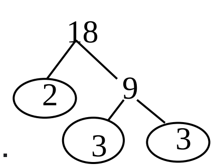
Step 2 Assist learners to write the circled numerals as $2 \times 2 \times 3$.
Step 3 Help learners to conclude that the prime factorisation of 12 is $2 \times 2 \times 3$

We can also go through similar steps to assist learners to find the prime factorisation of 18.

Step 1 Draw the factor tree of 18

Step 2 Product of circled numerals is $2 \times 3 \times 3$.

Step 3 Prime factorisation of 18 is thus $2 \times 3 \times 3$.



Key Ideas

- Even numbers are natural numbers with two as a factor.

- Odd numbers are natural numbers without two as a factor.
- Composite numbers are natural numbers with more than two factors.
- Prime numbers are natural numbers with only two factors; one and the number itself.
- Prime numbers can be generated using the **Sieve of Eratosthenes.**
- Prime factorisation of any natural number is the product of the prime numbers which are factors of the natural number.
- Use of factor tree facilitates the processes of finding the prime factorization of numbers.

Self-Assessment Questions

1. Describe one activity you would use to guide learners to distinguish between even and odd numbers.
2. What concept would you introduce to learners before the concept of prime numbers?
3. Describe how you would lead your Basic 5 learners to identify the following numerals as composite, deficient, and abundant number(s): 36, 49 and 57.
4. How would you explain to a Basic 6 pupil that 139 is not divisible by 2 (without performing the actual division) exactly?
5. How would you explain to a learner in Basic 6 that a given three – digit number is divisible by 2?

SESSION 3: FINDING HIGHEST COMMON FACTOR (HCF) AND LOWEST COMMON MULTIPLE (LCM)

The *Highest Common Factor* (*HCF*) or the *Greatest Common Factor* (*GCF*) of two or more given counting numbers is the greatest number that can divide the given numbers leaving no remainder. To help learners find the HCF of two numbers, the idea of intersection is used.

The *Lowest Common Multiple (LCM)* of two or more given counting numbers is the least number that the given numbers can divide leaving no remainder. Draw learners' attention to the fact that the *LCM* is the smallest natural number that is divisible by the given numbers. To help learners find the *LCM* of two numbers, the idea of union is used.

Learning outcomes

By the end of this session, you should be able to help learners to:

1. Find the Highest Common Factor (*HCF*) of two natural numbers
2. Find Lowest Common Multiple (*LCM*) of two natural numbers

3.1 Finding Highest Common Factor (HCF) of two natural numbers

For example, to guide learners to find the HCF of 12 and 18, learners should be guided through the following steps:

Step 1: Guide learners to write the factors of the two natural numbers:

Factors of 12 = 1, 2, 3, 4, 6, 12

Factors of 18 = 1, 2, 3, 6, 9, 18

Step 2: Assist learners to find the common factors.

Hence, the common factors = factors of 12 $\cap$ factors of 18

= 1, 2, 3, 6

Step 3: Lead learners to write down the highest or the greatest factor among the common factors

Among the common factors, the greatest is 6.

Step 4: Help learners to conclude that the HCF of 12 and 18 is 6.

Example 2

Going through a similar activity you can help learners to find the *HCF* of 24 and 36.

Solution

List the factors of the two numbers:

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18, 36

Therefore, factors of 24 $\cap$ factors of 36 = 1, 2, 3, 4, 6, 12

Among the common factors, the greatest is 12. Hence the HCF of 24 and 36 is 12.

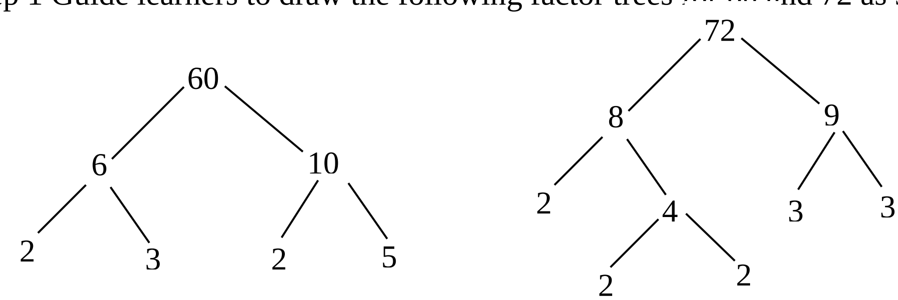
Another approach to find the *HCF* (or *GCF*) of, especially, large given numbers is to use *prime factorization.* In using the prime factorization, the *factor tree* may be used.

Example 1

Use prime factorization to find the HCF of 60 and 72.

Solution

Step 1 Guide learners to draw the following factor trees for 60 and 72 as shown below:



Step 2 using the factor trees, let learners express each number as the product of its prime factors

Thus, $60 = 2 \times 2 \times 3 \times 5$ $\quad\quad$ $72 = 2 \times 2 \times 2 \times 3 \times 3$

Step 3 Lead learners to discover all the common prime factors of both numbers ie. 2, 2, & 3

Step 4 Assist learners to conclude that $2 \times 2 \times 3 = 12$ is therefore the HCF/GCF of 60 and 72.

3.2 Finding *LCM* of two natural numbers

(a) Using the multiples and union approach
To find the *LCM* of two given numbers, we lead learners to:
Step 1: First list the multiples of each of the given numbers
Step 2: Then list the common multiples of the two sets of multiples of the numbers

Step 3: Finally, select the least of the common multiples as the *LCM*

Let us use the steps above to assist learners to work some examples.

Example 1
Find the LCM of 3 and 7.

Solution
We first list the multiples of 3 and 7
Multiples of 3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, ….
Multiples of 7 = 7, 14, 21, 28, 35, 42, 49,...
Common multiples = 21, 42, 63, …. (Multiples of 21)
Therefore, the LCM of 3 and 7 is 21.

Example 2
Find the LCM of 8 and 12.

Solution
Again, we list the multiples of 8 and 12.
Multiples of 8 = 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, …
Multiples of 12 = 12, 24, 36, 48, 60, 72, 84, ….
Common multiples = 24, 48, 72, … (Multiples of 24)
Therefore, the LCM of 8 and 12 is 24.

(b) Using prime factorisation and union method
Let us now consider finding the *LCM* of 8 and 12 using prime factorisation and union method.
First, you have to assist learners to find the prime factorisation of each number as follows:
Prime factorisation of 8 is $2 \times 2 \times 2$
Prime factorisation of 12 is $2 \times 2 \times 3$
The union of the two prime factorisations is $2 \times 2 \times 2 \times 3$
Assist learners to discover that $2 \times 2 \times 2 \times 3$ contains both the prime factorisation of 8 and 12.
Hence the LCM of 8 and 12 is $2 \times 2 \times 2 \times 3 = 24$

You can in addition guide learners to follow the steps below to find the LCM of 120 and 180:
Step 1: Prime factorisation of 120 is $2 \times 2 \times 2 \times 3 \times 5$
Step 2: Prime factorisation of 180 is $2 \times 2 \times 3 \times 3 \times 5$
Step 3: Assist learners to write $2 \times 2 \times 2 \times 3 \times 3 \times 5$ as the prime factorisation that has in it the prime factorisation of 120 and the same time 180 (union)

Step 4: Help learners to conclude that the LCM of 120 and 180 is $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

Key Ideas

- The HCF of the natural numbers X and Y is the highest of the common factors of both X and Y.
- The LCM of the natural numbers X and Y is the least of the common multiples of both X and Y.
- Prime factorisation of two or more natural numbers can be used to find the HCF (using the idea of intersection) and LCM (using the idea of union).

Self-Assessment Questions

1. Describe how you would use the factors and intersection approach to help Basic 6 learners to find the HCF of 45 and 63.
2. Describe how you would use the prime factorisation and intersection approach to help Basic 6 learners to find the HCF of 45 and 63.
3. Describe how you would assist a B6 learner to find the LCM of 45 and 60 using:
   (a) the multiples and union approach
   (b) Prime factorisation and union method
4. Describe how you would assist a B6 learner to find when two church bells, A and B which ring at intervals of 12 hours and 15 hours respectively would ring together again after Saturday 5.00am that they rung together.

UNIT 4: TEACHING RATIO, PROPORTION, PERCENTAGE AND MEASUREMENT

The understanding of ratios, proportion, percentages, and measurement will be of immense help to learners' day to day activities. This units introduces students to how to help learners to understand the concepts of ratio and proportion, percentages, measurement of length, area, capacity and volume, mass, and time.

Learning outcomes

By the end of this unit you will be able to teach your learners:

1. Concept of ratio, proportion, percentage, and their applications
2. Measurement of length, area, volume, and capacity
3. Measurement of mass and time

SESSION 1: CONCEPTUALIZING RATIO, PROPORTION, PERCENTAGE, AND THEIR APPLICATIONS

Ratio is one of the most useful ideas in everyday mathematics. The equality of ratios is called **proportion.** Percentages have been the language of society when it comes to among others education, trade, commerce, health, and science.

Learning outcomes

By the end of this session, you should be able to guide learners to:

1. develop the meaning of ratio and apply it in solving problems
2. discover the meaning of proportion and apply it in solving problems
3. discover the meaning of percentage and apply it in solving problems.

1.1 Developing the concept of ratio

The result of the question "By *how many is* 20 *greater than* 16?" is illustrated in Fig.4.1 above. There are 20 objects that are circled as group A. When 16 (circled as group B) is taken away, 4 is left.

We can compare two numbers or quantities by finding *how many times one is greater than/lesser than the other*? The response to this problem will lead to finding the ratio of the two quantities.

A **ratio** is a pair of positive numbers that is used to compare two quantities or two sets. The idea of ratio is illustrated in the Fig.4.1 below, which shows that for every 3 chips, there are 4 tiles. This ratio is written as 3 : 4 (read ratio of "3 to 4") or as the fraction $\frac{3}{4}$.

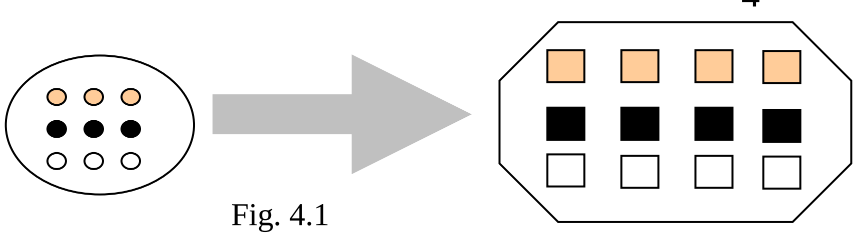


Fig. 4.1

Let learners compare two numbers or quantities by finding how many times one is as many as the other and write this as ratio. Example, 9 is one and half times as many as 6 because $\frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$.
This is illustrated in Fig.4.2 where members of the first set are compared with members of the second set.

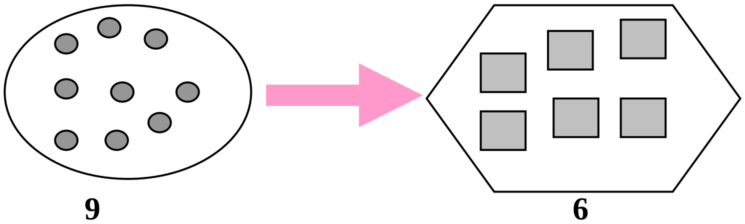


Fig. 4.3

Thus we write this ratio as 9 : 6
Work more examples of this with learners and assist learners to discover that ratio is about how **many times** more or less *and* not how **many more or less** or how **much more or less**. To get learners to answer the question ‘How many times more is Kwame who is 30 years older than Efua who is 10 years old?’ you simply get learners to express 30 years as a ratio of 10 years and conclude that the answer is 3 times.

Simplifying the ratio of two numbers or quantities
After finding ratios of many pairs of quantities or numbers, assist learners to simplify the ratios by expressing the ratio as a common fraction $\left(a\,;b = \frac{a}{b}\right)$, simplify it and rewrite the result as ratio. For example the ratio 9: 6 can be simplified to get 3 : 2 since $\frac{9}{6} = \frac{3}{2}$. Thus, the ratio 9 : 6 when simplified gives a ratio of 3 : 2.

Let learners find the ratio of the people in the first picture to those in the second picture, Fig. 4.4

Picture 1

Picture 2

Fig. 4.4

Can we simplify this ratio?

Work more examples with learners.

1.2 Discovering the meaning of proportion and apply it in solving problems
We may now assist your learners to discover proportion as equal ratios. You now lead learners to identify pairs of ratios which are equal. You may next assist your learners to express each pair as a proportion statement.

Finding missing numbers in equal ratios
Learners need to be led to discover that two ratios are equal if they have the same value when simplified.
For example the ratios 2 : 3 and 4 : 6 are equal since 4 : 6 when simplified gives 2 : 3.

For example, you can lead learners to find which pairs of ratios are equal:
(a) 6 : 8 and 12 : 18
(b) 18 : 36 and 25 : 50
(c) 30 : 40 and 90 : 120

Solutions
(a) Assist learners to simplify 6:8 completely as 3 : 4 and 12 : 18 completely as 2 : 3
Lead them to conclude that since 3 : 4 is not the same as 2 : 3, the pair of ratios 6 : 8 and 12 : 18 are not equal.

(b) Assist learners to simplify 18 : 36 completely as 1 : 2 and 25 : 50 completely as 1 : 2
Lead them to conclude that since 1 : 2 is the same as 1 : 2, the pair of ratios 18 : 36 and 25 : 50 are equal.

(c) Similarly, assist learners to simplify 30 : 40 completely as 3 : 4 and 90 : 120 completely as 3 : 4. Help them to conclude that since 3 : 4 is the same as 3 : 4, the pair of ratios 30 : 40 and 90 : 120 are equal.

Learners, well-grounded with understanding of equal ratios will find it easier to determine a missing number in an equal ratio expression. Let us consider some examples.
Learners may now be helped to find an unknown in a proportion sentence.

Example 1
Find the value of x given that $4 : x = 12 : 18$.

Solution
You can lead learners to solve this question by (a) inspection and (b) simplification approach.

(a) By inspection, learners are made to discover that 4 multiplied by **3** gives 12. Learners are then lead to conclude that since **3** multiplied by 6 gives 18 then x is 6.
(b) Learners can also be led to simplify 12 : 18 as 4 : 6. Thus, equating 4 : x to 12 : 18 is the same as equating 4 : x to 4 : 6. Hence, $x = 6$.

Applying the idea of proportion to share quantities

We can divide a quantity or a number into a given ratio. For example, to divide 15 oranges in the ratio “2 to 3”, we first find the total number of parts into which 15 oranges are to be divided. i.e. 2 + 3 = 5. For the first number: we find, $\frac{2}{5}$ of 15 which is 6. Similarly, the second number is $\frac{23}{5}$ of 15 which is 9. Therefore, dividing 15 oranges in the ratio 2 : 3 is “6 oranges to 9 oranges”.

This problem can also be looked at as two friends, Ama and Kojo, sharing 15 oranges in a ratio of 2 : 3 respectively.
To do dis practically, Ama and Kojo take turns to pick the 15 oranges 2 and 3 respectively at a go till all the oranges get finished. Each will count and tell how many oranges he or she had. Ama is expected to say 9 oranges and Kojo, 6 oranges.
Learners can be lead to do this activity in class using counting objects such as bottle tops instead of real oranges.

1.3. Discovering the meaning of percentage and applying it in solving problems

(a) Discovering the meaning of percentage
Percent, represented by the symbol %, simply means out of 100. Thus one-percent (written as 1% means 1 out of 100 or $\frac{1}{100}$.
We have already discussed common fractions in a previous session. We saw that a common fraction can be described as a ratio of two numbers. For example, the ratio 2 : 5 is the same as $\frac{2}{5}$.
To teach renaming of a common fraction in percentage, we begin with halves, fourths, and tenths. In other words, find *hundredths* and *percentage names* that are equal to halves, fourths, and tenths.

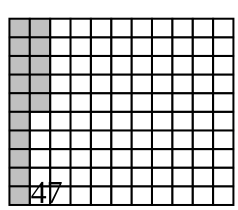
For example:

(i) $\frac{1}{2}=\frac{1}{2}\times\frac{100}{100}=\frac{50}{100}$ .This is written as 50 percent or symbolically 50%.

(ii) $\frac{1}{4}=\frac{1}{4}\times\frac{100}{100}=\frac{25}{100}=25\%$

(iii) $\frac{3}{4}=\frac{3}{4}\times\frac{100}{100}=\frac{75}{100}=75\%$

Hence we rename $\frac{1}{2}$ in percentage as 50%; $\frac{3}{4}$ as 75 % etc.
Use of diagrams are one method of gaining an understanding of percent. A 10 × 10 square grid with 100 equal unit squares is one of the common models for illustrating percentages or decimals. For example, Fig.4.5 shows 15%.



(15%)

Fig. 4.5

Now to change a fraction to percent, we can use the method of equal ratios with the second number of the second ratio as 100.

For example, to rename $\frac{2}{5}$ in percent, we write 2 : 5 = Δ : 100 Therefore

$$\frac{2}{5} = \frac{\Delta}{100} \Leftrightarrow 5\Delta = 2 \times 100 \Rightarrow \Delta = \frac{2 \times 100}{5} = 40$$

Hence the percentage name for $\frac{2}{5}$ is 40%. Work more examples with learners.

This approach can also be expressed as:

$$\frac{2}{5} = \frac{2}{5} \times 1$$

$$\frac{2}{5} = \frac{2}{5} \times \frac{100}{100}, \text{ since } 1 = \frac{100}{100}$$

$$\text{Thus, } \frac{2}{5} = \frac{2}{5} \times \frac{100}{100} = \frac{2}{5} \times 100\%$$

$$\frac{2}{5} = 40\%$$

Key Ideas

- The ratio of two numbers or two quantities can be determined by finding how many times one is as many as the other.
- Ratios can be applied in sharing or dividing quantities.
- Equality of ratios is proportion.
- Idea of proportion is used to solve real life problems where an increase in one quantity results in an increase in another quantity. The reverse is true.
- Common fractions with denominators one hundred (100) are called percentages.
- They are represented by the symbol %, which means ‘per hundred’.
-

Self-Assessment Questions

1. How would you explain to a B6 learner the meaning of 2 : 3.
2. Describe how you would assist a B6 to share 20 oranges to two of his friends, Ama and Kojo in the ratio of[C1] 1: 3, practically.
3. Using ‘by inspection method’, show step by step how you would assist a learner to find the value of m in the proportion $2 : m = 6 : 12$.
4. Draw to show 25%.

Self-Assessment Questions

1. Describe how you would introduce B5 learners to the concept of ratio.
2. Two friends, Malik and Musa are to share 20 chocolate pebbles in the ratio of 2:3 respectively. Describe how you would practically assist them to share.
3. The length of one stick is 2m longer than the length of the other. If the ratio of their lengths is 1:3, show how you would assist a B6 learner to find the lengths of the two sticks.
4. Percentages are fractions with denominators ……………………
5. Name one approach that you would adopt to help learners understand the concept of percentage.
6. Name one method you would use to guide learners change fractions to percentages.
7. In teaching learners decimals, we use the flat of Dienes' base-ten block to represent ….
8. Name the three categories that calculations with percentages fall.
9. How would you guide B5 learners to represent 0.7 as a percentage?

SESSION 2: TEACHING THE CONCEPTS OF MEASUREMENT OF LENGTH, AREA, VOLUME, AND CAPACITY

In this session we shall examine some ways of introducing measurement of length, area, and volume/capacity to learners. We shall first go through procedures of direct comparisons and indirect comparisons leading to the use of non-standard units. We shall then introduce the standard unit, and discuss some activities we may engage learners in such as estimations and measurement with standard instruments.

Learning outcomes

By the end of this session, you should be able to guide learners to:

1. compare lengths, heights, and distances by direct and indirect comparisons
2. compare surfaces of plane shapes by direct and indirect comparisons
3. Compare volumes/capacities by direct and indirect comparisons

2.1 Comparing lengths, heights, and distances by direct and indirect comparison

We introduce the concept of length to learners by asking them to compare two objects and make a perceptual decision about their lengths, width, thickness, and so on. Here, we guide the learners to bring the objects together on the same flat base and compare their lengths, heights, or thicknesses with reference to the common base. By this, we develop awareness of differences between the lengths, heights, or thicknesses of the objects.

As we engage the learners in the activities involving comparison, we introduce them to the use of the language of comparison in connection with the experiences. For example, in comparing the lengths of their pencils, Ama says "My pencil is longer than Kofi's''. The learners can also compare their heights and say, "Mensa is taller than Akosua'' "Akosua is shorter than Mensa" or "Mina is as tall as Abu". The activities can be extended to the ordering of three or more objects where superlative vocabulary is introduced. For example, we may lead the learners to say, "Kofi is the tallest pupil in the class".

We now introduce the idea of indirect measurement to the learners. We create the need for indirect measurement by asking learners to compare the lengths of two objects which cannot be brought together on the same base to make a perceptual decision regarding their length. For example, we can ask learners to compare the width of a window and the width of a door. To do this, we guide the learners to use some unit of measure. They may use parts of their body such as their arms, feet, hands, or some objects such as pencils, pens, or chalkboard erasers, among others. We match the unit of measure (sometimes called an arbitrary unit) with the length to be measured and determine how many times the unit of measure covers the entire length.
In this way the learners can determine the lengths of objects which cannot be brought together, and thus, tell which is longer or longest.

2.2 Comparing surfaces (areas) of plane shapes by direct and indirect comparison

(a) Comparing surfaces (areas) of plane shapes by direct comparison
Learners are led to place one surface on top of another and tell which of two surfaces is smaller or larger. They can do this easily if one surface or region fits within the other. We should encourage learners to use the language of comparison that goes with the activities. For example, a child may say that the top of the teacher's table is larger than the cover of her exercise book.
The learners may also order three or more shapes according to the sizes of the surfaces. We then lead them to tell which of them is the smallest or largest.

(b) Comparing surfaces (areas) of plane shapes by indirect comparison
Sometimes we may not be able to compare surfaces by placing one object surface on top of the other. In such cases arbitrary measuring units such as postage stamps, playing cards, packets, books, match boxes, postcards, cut-out squares, rectangles and triangles may be used to compare their areas.

Initially, the surfaces the learners' measure should be such that they take whole number of units. We should also give opportunities for them to experience situations where gaps are left over along the boundary after covering the surface with units. We may then guide them to cut the arbitrary unit into halves and fourths, or into other fractional parts and use these to measure the area of the gaps.

For example, the area of a rectangular surface can be measured using old postage stamps of the same type as arbitrary units as shown in Fig. 4.9.

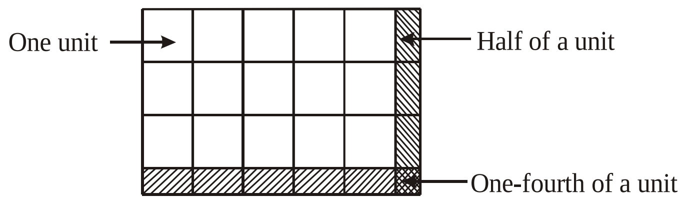


Fig. 4.9

The single-shaded portions indicate gaps which require half-units to cover, while the double-shaded portion requires one-fourth unit. Thus, the rectangular surface measures 15 units (whole stamps), 8 half-units (half stamps) and 1 one-fourth unit (one-fourth of a stamp) giving altogether 19¼ units as the area.

We should give learners the opportunity to cover surfaces using a variety of arbitrary units and determine their areas as shown above. Normally, learners measure rectangular and square shapes using small rectangles and squares as units. However, in order that they do not get the impression that only rectangles and squares have area they should also measure areas of other shapes. They may have problems measuring the areas of these shapes. We should direct them to cover most of the shapes with whole units and then estimate how many more units they require to cover the gaps left along the sides.

Activities using a geoboard are useful in giving learners experience for the understanding of area. A geoboard (also called nail board) is a device involving a square-shaped piece of wood with nails driven in at equal intervals throughout (preferably $1cm^2$ intervals). An example of a geoboard is shown in Fig 4.10.

Elastic bands are used to make different shapes by passing them round the nail heads. Learners make regular and irregular shapes of their own and find their areas by counting the unit squares enclosed by the elastic band which forms the shape.

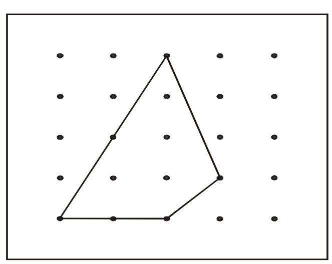


Fig. 4.10

Activities involving the use of squared paper or graph sheet are also helpful. Learners trace the outlines of objects (such as erasers, match box, leaves and so on) on the squared paper and count the unit squares enclosed to determine the areas occupied by the objects. Where the outline does not cover a whole square, learners are guided to count every unit square which has half or greater part of it within the boundary as one whole square but should not count it if less than half of it lies within the boundary. Thus, learners will have much experience determining the approximate areas

of shapes. Regular and irregular shapes with straight sides are also drawn on the squared paper and learners count the squares enclosed to determine their areas.
Learners can also be given several 1 cm^2 cut out cardboards and asked to cover surfaces (surfaces of exercise books and textbooks) with them to determine their areas in cm^2.

Introduce 1 m^2 manila card or cardboard to children. Get them to estimate areas of surfaces (the surface of the teacher's table and the floor of their classroom). Assist the learners to use the 1m^2 cardboard to find the area of the surfaces. Encourage learners to record their answers in short form as example, 20 m^2 (twenty metre square).

2.3 Comparing volumes/capacities by direct and indirect comparisons

Capacity is how much a container can hold or amount of space and volume is about the space occupied by an object. We introduce capacity to young learners by asking, "Which container holds more?"
We can guide learners to make perceptual comparisons between two containers to determine which of them holds more. We must note, however, that young learners often make the comparisons of capacities of containers based on sizes or heights rather than on their capacities. When asked which holds more, a tall container, or a short container, most learners will choose the tall container even if the short one can hold more water. Thus, it is probably best to begin the study of capacity by using direct comparisons.
We can help learners to expand their knowledge of volume in a variety of ways. They may build with small and large blocks. They may also fill empty containers with water or sand and find how much water or sand a container can hold. Learners may, equally fill empty containers with a variety of materials (units) such as seeds, bottle tops, or wooden blocks to determine their volumes.

(a) Comparing the capacities of containers and to determine which holds more
We give a variety of containers to learners and guide them to fill one with water or sand and pour it into the other to see which holds more. After learners have done some comparison on their own, we guide them through more structured activities. We can have learners guess which of two containers holds more, and then check the results with them. The learners can share findings. For example, they may say 'the milk tin holds less than the milo tin or 'the milo tin holds more than the milk tin'.

(b) Using arbitrary units to measure the capacities of containers
We use indirect comparison when two containers cannot be compared perceptually or directly. Suppose we have two containers with small openings that make it difficult to pour from one into another. We help learners to pour water or sand from each into identical containers with wide openings and then compare them to determine which holds more or the most.

We also introduce the measurement of capacity using arbitrary units. We can use small cups, tins, bottles as arbitrary units to measure the capacities of large containers. Here the learners find the number of times the smaller container can fill the large one. Learners compare the capacities of containers using the smaller containers as arbitrary units.

(c) Measuring the capacities of containers in litres

We introduce the litre as the unit for most liquid measures. We show learners bottles or containers which can hold a litre of water or any other liquid. They examine them and use them to measure the capacities of large containers. We should encourage them to estimate first and then check by measuring. We should introduce "l" for litre and encourage them to record measurement of capacities using this symbol.

(d) Determining the volume of different containers using arbitrary unit

We ask learners to collect many bottle tops of the same variety. We then provide them with various containers: milk tin, chalk box, drinking cup and so on. We ask learners to estimate and then determine the volume of each container in terms of the number of bottle tops. They order the containers based on their findings.

As an extension of this activity, we select another unit, such as dried seeds, and repeat the procedure. The learners will notice that the results of the ordering will be the same as in the first case.

Key Ideas

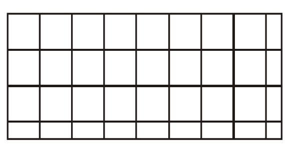
- Measuring of quantities such as length, area, and volume/capacity begin with direct comparison.
- In situations where direct comparison cannot be used to compare quantities, indirect comparison involving the use of non-standard units (arbitrary units) of measure are used.
- The use of different arbitrary units to measure a given quantity result in obtaining different results.
- A standard unit is used in the measurement of a given quantity to ensure the same result is obtained.

Self-Assessment Questions

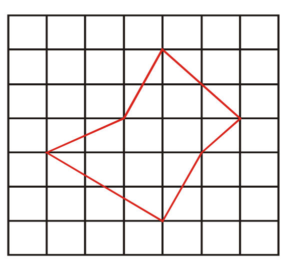
Indicate whether the following statement is true or false:

1. We introduce a standard unit because learners do not know how to measure with non-standard units.
2. Kofi and Aba each measured the length of the class 3 teacher's table using their hand spans. Kofi had 10 while Aba had 13 hand spans. Which of the two measurements is acceptable? Why?
3. Fill in the blank space: The basic unit of measurement of length is the …………………
4. Indicate the unit of measure (metre or centimetre) which you would use in measuring the length of each of the following:

   (a) A pencil
   (b) A class 1 pupil's writing slate
   (c) The length of the school football field
   (d) A line segment in a pupil's mathematics textbook
   (e) The length of the crossbar of a football goal post.
   (f) The height of a class 2 pupil's chair
5. How can a child who cannot measure using the conventional standard unit tell which of the front covers of two books has a bigger area?
6. A region on the floor of a classroom has been marked for a play-shop. A child compares this region with the top of the teacher's table. How can she determine which of the regions is larger if she cannot measure using the conventional units?
7. A primary class 3 pupil determined the area of a rectangular region by covering the region with identical postcards. She covered the gaps left along the sides with half-units and one-fourth unit of the postcard as shown in the diagram below. How would you guide a child to determine the area of the rectangular region?



8. How can a child find the area of the shape drawn on the squared paper below?



9. A primary class 1 pupil wishes to determine which of two containers is larger. How will you guide the child to do so?
10. A child wishes to find the capacities of several containers which cannot be brought together. How will she do this?

11. A pupil fully stacked a box 6cm long, 3cm wide and 4cm high with one-centimetre cubes. Briefly explain to the pupil the relationship that exists between the number of cubes in the box and the length, width, and height of the box.
12. The length, width and height of a rectangular box are 5 cm, 3cm, and h cm respectively. A child stacked the box with one-centimetre cubes and noticed that 45 of them filled it. How would you help the child to determine the height *h* cm of the box?

SESSION 3 MEASUREMENT OF MASS AND TIME

Mass and weight are often used interchangeably, but their meanings differ. Weight is related to the gravitational pull of the earth. It relates the action of force (gravity, for example) on a mass. Mass remains the same regardless of location.

Time is a difficult concept for learners to grasp. They cannot see it or feel it. It is the measure of a period between two events or during which something takes place. It is also a precise moment determined by a clock.

In this session we shall discuss some ways of introducing the measurement of mass to learners. We shall follow the same trend as that for the measurement of length. We shall also look at some ways by which we can help learners to have an idea of time. We shall discuss some activities we may involve learners in so that they can develop the awareness of passage of time.

Learning outcomes

By the end of the session, you should be able to guide learners to:
1. compare the masses of objects
2. use arbitrary units to measure masses of objects
3. identify some events and tell when they take place
4. identify events which take a short time and those which take a long time
5. find the duration between two events

3.1 Comparing the masses of objects

We guide learners to compare the masses of objects by handling them and thus, determine which is heavier in each case. Here also, the language of comparison is important. The activities should be extended to the ordering of more than two objects and the superlative vocabulary used appropriately.

We should give the learners the opportunity to discover for themselves that the size or bulk of an object may not necessarily imply that it is heavier than another. For example, they may discover by handling that a small stone is heavier than a big bag full of cotton wool.

Learners may also use the see-saw to compare their masses with other members of their class and order themselves according to their masses. We may again guide the learners to use a simple balance to determine which of two objects is heavier, or to order more than two objects.

3.2 Using arbitrary units to measure masses of objects

Once learners have had sufficient experience of comparing the masses of objects and ordering them by handling or using the see-saw and the simple balance, we can introduce them to the measurement of masses. We engage them in measurement activities in which they use the simple balance and appropriate arbitrary units to determine the masses of objects. Some of the arbitrary units they can use are seeds, beads, nails, cubes, pencils, erasers and exercise books. They measure the masses of all kinds of classroom objects to the nearest whole number of arbitrary units using the simple balance. Later, we guide them to use fractional parts of arbitrary units.

After comparison and ordering of masses, learners will notice that they need a more standard measure. We introduce the kilogram as the basic unit for the measurement of mass. We provide 1-kilogram masses, and each learner holds this mass in his hand to get the 'feel' of it. Then they pick up objects in the classroom or outside and determine whether they weigh about 1 kilogram or more than 1 kilogram or less. They check their results using a balance.

Learners are also encouraged to estimate the masses of objects in kilograms and verify their estimates by measuring with a balance. They estimate their masses to the nearest kilogram and check them on a scale. Learners then determine the heaviest child in the class and group themselves by their masses. They also compare their masses with their teacher's and calculate the difference between them.

We introduce the 'gram' as a unit of measure which is one-thousandth part of the kilogram. Learners are led to measure the masses of objects in grams using the simple balance. We also assist learners to weigh out sand, seeds or grains of 100 grams, 300 grams, 500 grams and so on, in bags. They can use these to determine the masses of other objects by balancing these masses with the objects. We should give learners the opportunity to determine the masses of objects through many activities involving the use of the simple balance.

We should also encourage learners to estimate the masses of small objects in grams and check these by measuring them on a simple balance.

3.3 Identifying some events and telling when they take place

For learners to understand the concept of the passage of time, we need to expose them to experiences in which they become aware of when an event takes place. The experiences should include activities to develop a sense of sunrise, breakfast time, time they go to school, time they close from school and sunset. They should also include a sense of yesterday, today, and tomorrow along with the ideas of morning, afternoon, evening and night, and the days of the week. The activities should also make the learners aware of continuation of time through days, weeks, and months.

Using the calendar, we can lead learners to find out the number of days left for special events such as a holiday, an occasion in their school, or school vacation. With a calendar available as a reference point, we can help learners build clear sense of the passage of time.

3.4 Identifying events which take a short time and those which take a long time

We need to help learners to develop awareness of time by helping them to responding to the question such as 'how long an event takes?'. We can have them compare the times of events to establish whether one event takes longer time or shorter time than another. Events may include clapping twenty times, bouncing a ball ten times, and emptying water from containers with different sizes plastic paint buckets. For example, in the last of these events, learners take several identical cans, and they are guided to make three holes in the bottom, using a nail of different size for each can. The learners examine the cans and guess which they think will empty first, second, third, and so on.

Using two cans at a time the learners pour equal amounts of water in each can and observe them as they begin emptying at the same time. They repeat this process until the cans have been ordered from the fastest to the slowest. Similarly, the cans may have a different number of holes all the same size or a different number of holes of varying sizes.

The problem of comparing events that cannot occur simultaneously or of indicating "how long" a single event takes, leads learners to the use of various units of time measure. Initially, we begin with the use of arbitrary units such as hand claps, steps, jumps, and so on, to measure the time it takes to complete simple events like singing a song or walking from the teacher's table in front to the back of the classroom.

We may also develop timing devices and use them as arbitrary units (Fig 4.12). We fill a funnel with sand. The amount of time for the sand to empty becomes an arbitrary unit of time.

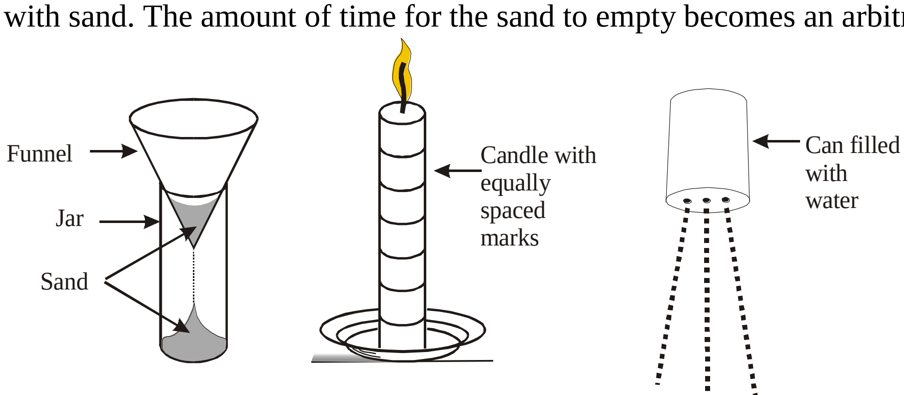


Fig. 4.12

Another timer we can develop is a candle on which several equally spaced marks have been made. The amount of time for the candle to burn from one mark to another becomes an arbitrary unit of time.

We can also construct a timer by making holes in the bottom of a tin/container and fill it with water. The count of drops for the tin/container to become empty (10 drops) gives the arbitrary time and the drops, an arbitrary units.

3.5 Finding the duration between two events

This may involve learners finding the number of months, weeks, days, hours, and minutes between two events. For example, if the market days in a town are Tuesday and Saturday, learners determine the number of days between one market day and the next. There are some market days that take place some days after a particular market day. For example, 5 days interval market days. By this, learners should be able tell when the next market day will take place. They may also determine the number of months, weeks, and days between the celebration of Ghana's Independence Day Anniversary in the year 2001 and the year 2002; or between the school's reopening day and the vacation day of the first term in a particular year.
Learners may also use the calendar to determine the duration between Christmas Day in a particular year and Easter Day in the following year.

Key Ideas

- Mass is about the heaviness or the lightness of an object.
- Mass is not the same as weight. It is measured in grams and kilograms.
- Weight is about force an object exerts on anything that freely supports it. It is measured in newton.
- A beam balance can be used in helping learners to compare masses.
- Heavy objects are usually measured in kilogram and light objects are usually measured in grams.
- Time is about a measure of the duration an event takes place.
- Time is usually measured in hours, minutes, and seconds.
- The analogue and digital clocks are used to measure time.

Self-Assessment Questions

1. Which one of the arbitrary units in brackets will you use to weigh?
    a. a pupil's maths exercise book? (Pupil's mathematics textbooks, erasers)
    b. 30 cm ruler? (Pencils, 30 – page exercise books)
    c. a Fanta bottle top? (Chalkboard dusters, one-inch nails)
2. Give one everyday-life activity you would use as an example to show learners where the kilogram is commonly used as a unit of measure.
3. Indicate if each of the following weighs about 1 kg, less than 1 kg, or more than 1 kg.
    a. a chalkboard eraser
    b. a packet of 6 pencils
    c. a metre rule
    d. a carton of 12 bars of key soap
    e. a cake of Lux soap
    f. a primary class 1 pupil
4. In which units of measure would you direct a child to estimate the masses of each of the following?
    a. a pencil
    b. the smallest pupil in primary class 1
    c. a 20-leaf exercise book
    d. a pile of 20 exercise books
5. A child who was asked to express the mass of an object given as 4 kilograms 2 grams in decimal notation put down 4.02 kg. How will you lead the child to find out whether he is right or wrong?
6. How will you help a child to find out what fraction of a day is an hour?
7. What does a child need to know to be able to find the number of hours in 150 minutes?
8. A child wishes to find out the number of days between September 26th and November 9th of last year. How will you help the child to do this? What teaching-learning material can he use?
9. Describe an activity you would use to guide a primary Basic 6 pupil to determine the duration in minutes and seconds between two events.

UNIT 5: TEACHING GEOMETRIC CONCEPTS AND NUMBER PLANE

The things we see round us are full of geometric shapes. Basically, they are three dimensional (solid shapes) with their faces being 2 dimensional (plane shapes). Geometric concepts such as points, lines rays, line, and angles will be treated in this unit. The number plane will be introduced and location of points as well as drawing of geometric plane shapes on the number plane will be discussed.

Learning outcomes

By the end of this unit you will be able to teach your learners the following:

1. Concepts of: points lines, line segments, and ray
2. Common plane shapes and their properties (polygons)
3. Common solid shapes (prisms and pyramids) and their nets
4. Number plane and plotting of points on the number plane

SESSION 1: POINTS, LINES, LINE SEGMENTS AND RAYS

Points, lines, line segments and rays are all geometric concepts. Learners can learn these concepts only if they experience it. Raw definition of each concept will not be of immense help to learners. Your illustration as a teacher by giving several examples using real objects will help learners to explain these concepts on their own.

Objectives

By the end of this session, you will be able to assist learners to:

1. define points
2. define lines
3. define line segments
4. define rays

1.1 Defining points

Space is thought of as being made up of locations or points. Geometric figures are thought of as sets of points. Here are some activities you can do in the class to help your learners to understand the concept of points.

- Point to the tips of objects in the classroom and have learners to identify them as points. E.g., Tip of a pencil/pen, tip of a book, tip of a table, etc.
- Take learners out and let them talk about points on objects outside their classroom. E.g., Tip of a stick, tip of an electric pole, tip of the roof of a building etc.
- Lead learners to discover that when 2 or more edges meet a point is formed.

Make a dot (an infinitesimal radius circle) on the board for learners to see. Point out to learners that a dot is used to represent a point geometrically. In addition to the representation, a capital letter of the alphabets is used to describe or indicate a point, e.g. **A** is described as a point.

A •

1.2 Defining lines

Your learners may be familiar with the term “a line” but may be lacking the concept of a line. A careful illustration of a line is therefore important. Let two learners hold the two ends of a twine/string respectively such that those in the class will **not** see the two learners but can see the twine/string passing through their classroom. (See Fig.5.1a)

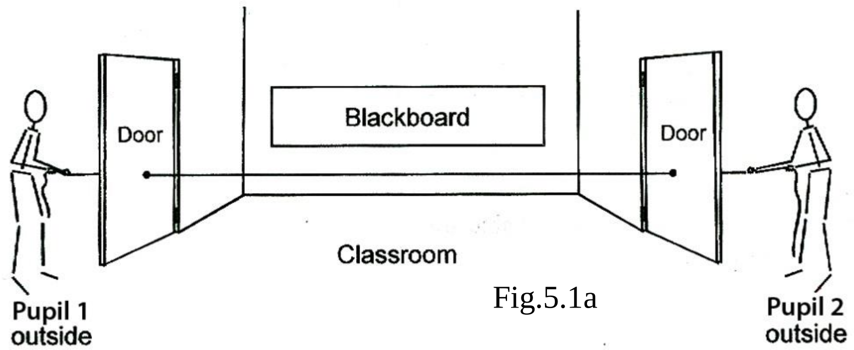


Fig.5.1a

Learners in the classroom will see the twine/string as endless. Also lead your learners to talk about the number of points on the line and assist them to conclude that there are several points on a line. With these discoveries, lead learners to conclude that a line has no end points and that it’s made up of several points which follow a straight path. Ask your learners to give examples of a line in this physical world. (High tension wire/the edge of a straight street).

Help learners name for example, a line as $\overleftrightarrow{AB}$ (read as line AB). Draw on the board to show a representation of a line *AB* (Fig.5.1b).

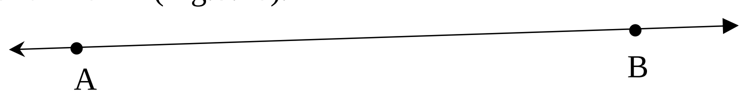


Fig. 5.1b

1.3 Conceptualizing a line segment

Let the two learners hold the ends of the twine/string in-front of the class, keeping string/twine straight. (See Fig 5.2)

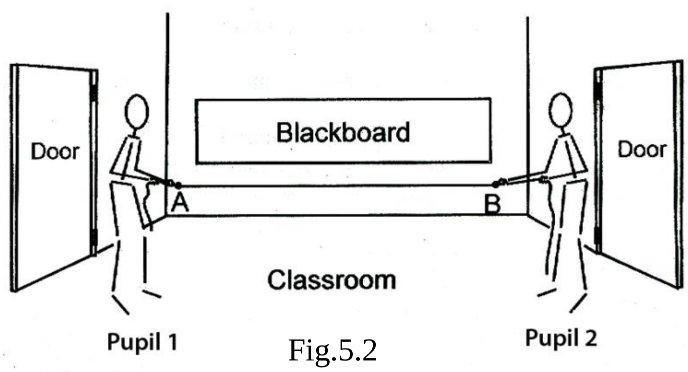


Fig.5.2

Call the two end-points A and B. Point out to learners that the line is now called a line segment AB, written as $\overline{AB}$. Ask learners to tell the number of points on the twine/string. Learners should conclude that there are uncountable points on a line segment.

Lead learners to identify edges of tables, the writing board, doors, and windows as examples of line segments.

Assist learners to conclude that a line segment is formed when two points are joined by a straight line. This means a line segment has two end points. Draw line segments of different lengths on the board using a writing board ruler. Let learners name the line segments.

1.4 Conceptualizing a ray

Make your classroom darker by closing the windows and doors. Switch on a torch light. Allow learners to observe the beams from the torch light. Open the windows and doors. Call a pupil to the board to sketch his/her observation.

Let two learners hold the twine straight again at the ends, let one person stand outside and the other inside the classroom. (see Fig 5.3)

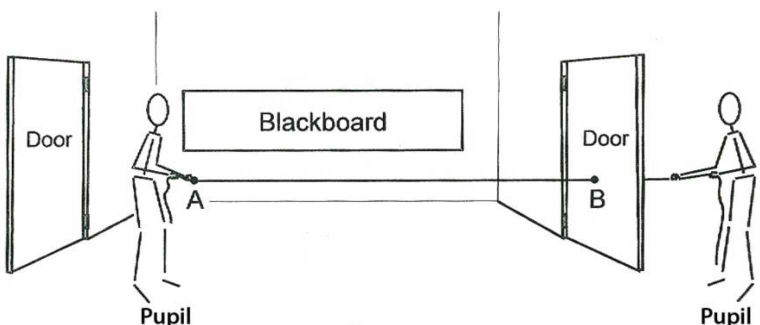
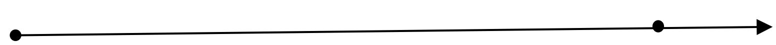


Fig. 3.3

Through discussion, assist learners to identify the new picture, this time as a ray. Assist learners to conclude that a ray has only one end point. Draw/sketch ray *AB* on the board as shown below. (See Fig 5.4)

B



B

A **Fig 5.4**

Write down the representation of the ray *AB* drawn on the board as $\overrightarrow{AB}$.

Draw several diagrams of rays, Fig.5.3, line segments on the board. Let learners identify the points, lines, line segments, and ray on the diagram in the drawings.

Discuss answers with learners and lead them to draw a clear distinction between the four concepts

Key Ideas

- Geometrical shapes are made up of points, lines, line segment, and rays
- A point is a non-dimensional circle.
- When two lines meet, a point is formed.
- A line segment has two end points.
- A line has no end points.
- A ray has only one end point.

Self-Assessment Questions

1. List 3 examples each in the physical world to illustrate:
   (i) a ray
   (ii) a line
   (iii) a line-segment

SESSION 2: COMMON PLANE SHAPES AND THEIR PROPERTIES

Plane shapes are basically flat closed surfaces. Learners are already familiar with flat surfaces. Solid shapes such as exercise books and textbooks can be used to teach the concept of plane shapes only if the illustrations are based on the surfaces but not on the entire solid shapes.

Learning outcomes

By the end of this session, you will be able to help learners to:

1. Identify common planes by their names
2. Describe plane shapes by the labels of vertices
3. Identify plane shapes with square corners

2.1 Identifying common plane shapes by their names

Run your palm over flat surfaces of objects in the classroom in turns. At each turn point out to learners that the surface is a plane. Ask learners to state the number of points on each surface and lead them to conclude that **a plane is a set of points suggested by a flat surface (eg**. Surfaces of table, white board, chairs, etc.). Let learners give examples of planes in their homes. Show cut out plane shapes to learners. Lead them to group these shapes into two main groups: circles and polygons. Get them to describe circles as flat surfaces bounded by perfect closed curves, and polygons as flat surfaces bounded by closed line segments.

Take the polygons and assist learners to sort them out into triangles (3-sided polygons), quadrilateral, (4-sided polygons), pentagons (5-sided polygons), hexagons (6-sided polygons), heptagons (7-sided polygons), etc.
Our discussion on flat figures will be on naming of triangles and quadrilaterals. We will also identify some basic properties of triangles and quadrilaterals.

(a) Triangles

Draw a triangle on the board using a metre rule or any straight edge. Label the points of intersection of pairs of line segments, *A*, *B* and *C*. Help learners to name the triangle as triangle *ABC*.

Show cut-out triangles to learners and lead them to identify the triangles as:

1. Right-angled triangle (a triangle with one angle being 90°, Fig 5.5)

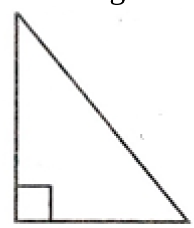


Fig.5.5

2. Equilateral triangle (a triangle with all sides or all angles being equal, Fig 5.6).

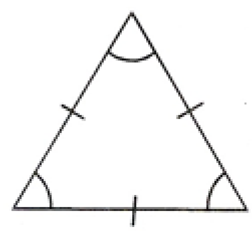


Fig. 5.6

3. Isosceles triangle (a triangle with two sides or two angles being equal, Fig (5.7).

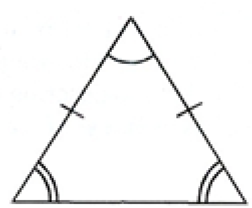


Fig. 5.7

4. Scalene triangle (a triangle with all sides being different and all angles also different in size (Fig 5.8).

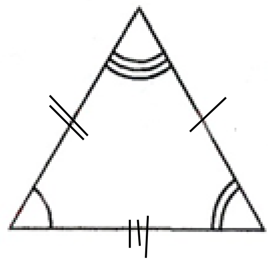


Fig. 5.8

Use folding or measuring activities to lead learners to discuss these properties on their own. With the help of straws, assist learners also to discover that a triangle can only be formed when the sum of the lengths of the two shorter sides is more or greater than the length of the longest side.

(b) Quadrilaterals

Draw any four-sided polygon on the board. Label the points of intersections of the line segments as *A, B, C, D*. (Note: order is important in labelling of quadrilaterals). Assist learners to name the quadrilaterals as quadrilateral *ABCD* (Fig. 5.9).

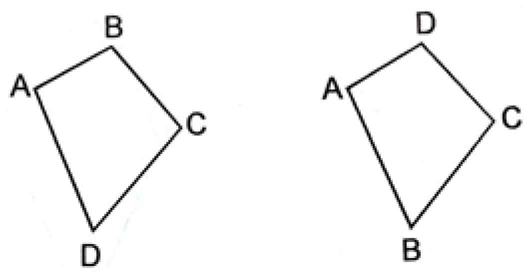


Fig 5.9

Give learners different cut-out quadrilaterals and help them to identify each quadrilateral by its name.

1. Square:- a quadrilateral with all sides equal and each interior angle being 90° (Fig.5.10).

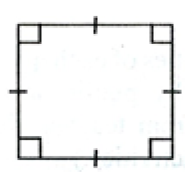


Fig. 5.10

2. Rectangle:- a quadrilateral with pairs of opposite sides equal and each interior angle being 90°, (Fig. 5.11).

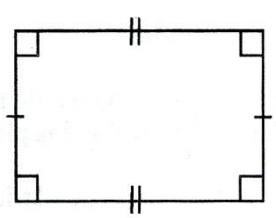


Fig. 5.11

3. Parallelogram:- a quadrilateral with pairs of opposite sides being parallel and equal and pairs of opposite angles also equal, (Fig. 5.12).

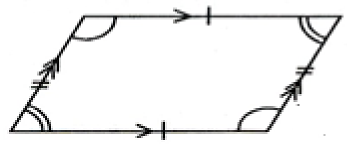


Fig. 5.12

4. Rhombus:- a parallelogram with all sides equal, (Fig. 5.13).

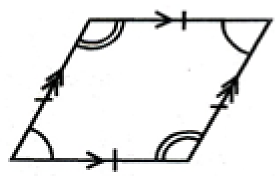


Fig 5.13

5. Kite:- A quadrilateral in which pairs of adjacent sides are equal and two opposite angles of the same size (Fig 5.14),

Fig 5.14

6. Trapezium: - a quadrilateral in which two opposite sides are parallel (Fig. 5.15).

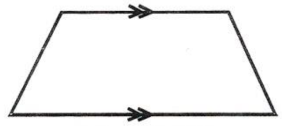
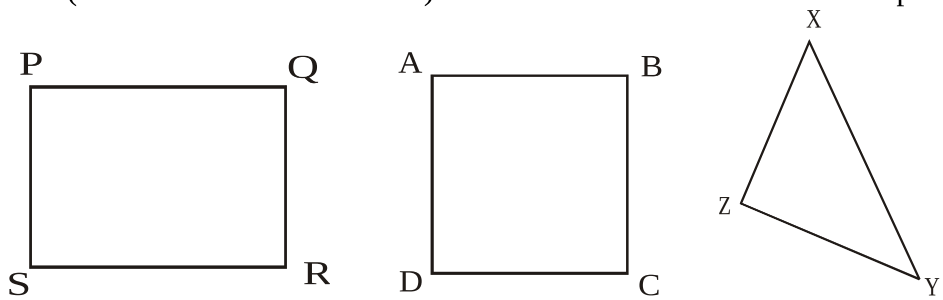


Fig 5.15

(NB. Properties of each quadrilateral should be discovered by learner or learners with little assistance from teacher, through folding and measuring activities).

2.2 Describing plane shapes by the labels of vertices

Assist learners to label the vertices of plane shapes with capital letters, (Fig.5.16). They read the letters in (clockwise or anticlockwise) order and use these to name the shapes.



Rectangle PQRS **Square ABCD** **Triangle XYZ**

Fig.5.16

The learners are also given a set of different shapes with the vertices labelled. They are then asked to name shapes of a certain description using the letters at their vertices. They also identify shapes whose names are called out.

2.3 Identifying plane shapes with square corners

We introduce the square corner and lead learners to use the corners of cut-out unit squares to test and find other square corners of plane shapes. For example, using a corner of a cut-out unit square, learners test to find out that the corners of books and sheets of paper are square corners. They also identify square corners on their tables and chairs, the chalkboard, the walls, and floor of the classroom and in other objects in the classroom and outside.

We name a square corner as a right angle and guide learners to identify plane shapes with right angles (square corners). Examples of these are the square and rectangles. They also identify the square corner in other shapes.

Key Ideas

- A plane is a set of points suggested by a flat surface
- There are different kinds of plane shapes. Some are bounded by line segments, others are bounded by line segments and curve(s) and others by a curve.
- Plane shapes bounded by three line segments are called triangles.
- Plane shapes bounded by four line segments are called quadrilaterals.
- Plane shapes bounded by several line segments including triangles and quadrilaterals are called polygons.

Self-Assessment Questions

1. Write down the properties of the following plane shapes:
   (a) Equilateral triangle
   (b) Isosceles triangle
   (c) Rectangle
   (d) Parallelogram.

2. A corner of a shape is called……………………… ..

3. What is the shape of a face of a rectangular pyramid, which is not the base?

4. Name the shape of the face exposed to view when you cut a sphere into two.

SESSION 3: COMMON SOLID SHAPES

Solid shapes can be put into three main groups: prisms, pyramids and spheres. Prisms have uniform cross-sections whiles pyramids have non-uniform cross-sections.
Studying the geometric properties of solid shapes provide learners with the opportunity to learn the process goals of geometry.

Learning outcomes

By the end of this session, you will be able to lead learners to:

1. classify common solid shapes into prisms, pyramids, and spheres
2. name prismatic solid shapes
3. name pyramidal solid shapes
4. make a free hand sketch of common solid shapes

3.1 Classifying common solid shapes into prisms, pyramids, and spheres

Set before the class, collections of empty containers, cans, and cardboard models of solids of various sizes for learners to observe, compare and sort them into the three main groups: Prisms, pyramids, and spheres.
Discuss with learners the features of each group. Lead learners to come out with the following features.

(a) Prisms
Prisms are solid shapes which have uniform cross-sections. They are made up of **pile of congruent plane shapes**. Some have polygonal uniform cross-section and others circular uniform cross-section.

(b) Pyramids
Pyramids are solid shapes with non-uniform cross-sections. Pyramids have slant edges and all the slant edges of the pyramid converge at a common point (the vertex). Pyramids have polygonal bases or circular bases. They are **made up of a pile of similar planes shapes**. The slant edges of the pyramids converge at a common point called vertex (apex).

(c) Spheres
They are round solid shapes.

> **Your learners at this stage should be encouraged to give examples of real objects which are prisms, pyramids, and spheres.**

Prisms: Key bar soap, empty tins such as milk tins, milo tins, empty boxes or cartons, ludo die, unsharpened pencil, etc.
Pyramids: Christmas hat, heap of sand, Egyptian Pyramid, roof of summer hut, etc.

Spheres: tennis ball or football, globe, shot put, ball bearing etc.

3.2 Naming prismatic solid shapes

Set the various prismatic shapes before the class. Give learners the opportunity to describe the shape of the cross-section of each prism or name the cross section. Some of the prisms have circular base, some have triangular cross-section, some have square or rectangular cross section while some have pentagonal cross-section, etc. Lead learners to name the prismatic shapes with:

1. Circular cross-section as circular prisms or cylinders. (Fig 5.17)

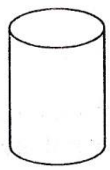
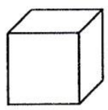
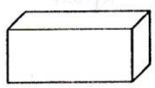


Fig. 5.17 Cylinder

2. Square or rectangular cross-section as cubes or cuboids. (Fig 5.18 and Fig 5.19) (A cube has all the faces to be squares whereas a cuboid may have some faces as squares and others as rectangles).



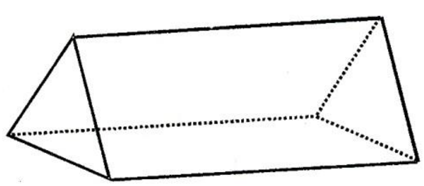
Cube
Fig. 5.18



Cuboid
Fig. 5.19

Note also that all cubes are cuboids.

3. Triangular cross-section as triangular prisms (Fig 5.20)



Triangular prism
Fig 5.20

4. Pentagonal cross-section as pentagonal prism (Fig. 5.21)

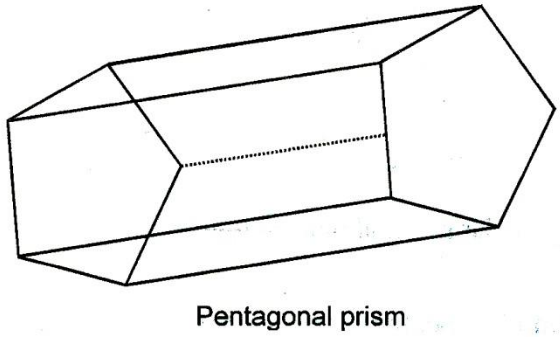


Fig. 5.21

Finally, let learners make a free hand drawing or sketches of some of the common shapes. Encourage learners to use ruler where necessary.

3.3 Naming of pyramidal solid shapes

Display models of pyramidal shapes on your table. Give learners the opportunity to examine the base of each shape. Let them mention the name of the shape of the base of each shape. By this action, assist learners to name the pyramids as follows.

1. Circular-base pyramid as circular pyramid or cone. (See Fig. 5.22)

Fig. 5.22

2. Square/rectangular-base pyramid as square or rectangular pyramid (Fig. 5.23)

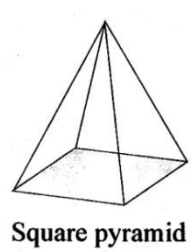
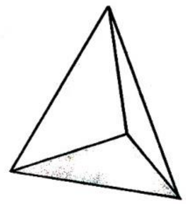


Fig. 5.23

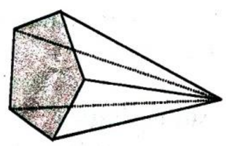
3. Triangular-base pyramid as triangular pyramid or tetrahedron (Fig. 5.24).



Triangular pyramid

Fig. 5.24

4. Pentagon-base pyramid as pentagonal pyramid (Fig 5.25)



Pentagonal pyramid

Fig 5.25

3.4 Making free hand sketches of solid shapes

With the help of circular objects and straight edges (ruler) have learners to make free hand sketches of these shapes in their exercise books and label them.

Key Ideas

- Solid shapes are three dimensional (3-D).
- Solid shapes can be put into three main groups: prisms, pyramids and spheres.
- Prisms have uniform cross-sections
- Pyramids have non-uniform cross-sections.
- Apart from the cylinder, and the cross-sectional faces, prisms have rectangular faces.
- Apart from the cone, pyramids have triangular faces.
- Pyramids have slant edges which converge at common point called vertex.

Self-Assessment Questions

- Describe how you would assist your B6 learners to make out the difference between a pyramid and a prism.

- Explain why a cube is a cuboid but not all cuboids are cubes.

SESSION 4: THE NUMBER PLANE AND PLOTTING POINTS ON THE NUMBER PLANE

We may represent data on a grid using points. For learners to be able to understand such data, therefore, we should teach them to represent and interpret the data correctly. In this session we shall discuss how learners can helped to locate points on the number plane or plot pairs of numbers on a grid to represent points on a number plane.

Learning outcomes

By the end of this session, you should be able to guide learners to:

1. use numbers for columns and rows to locate positions of objects in a rectangular array;
2. find ordered pairs of numbers for points on the number plane; and

4.1 Locating positions using columns and rows

We arrange objects in a square grid in columns and rows. We assign numbers to the columns and rows as column 1, column 2, row 1, row 2 and so on.

We then guide learners to locate the positions of objects using the column and row numbers. For example, in Fig. 5.31, we locate the object P in column 3 and row 4. That is, P is in column 3, row 4 or (3, 4).

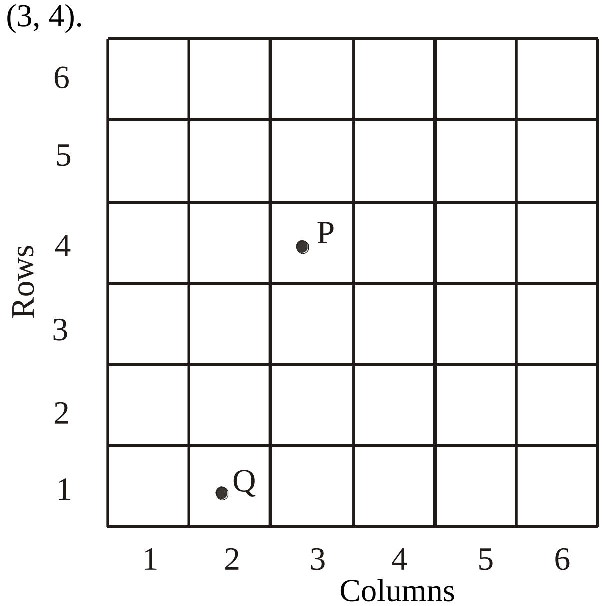


Fig. 5.31

Learners are asked to identify the location of object Q. Learners are assisted to practice plotting object points on a grid paper or a graph sheet and record the positions as (column x, row y) or simply (x, y).

4.2 Points on the Number Plane

We guide learners to draw a horizontal line and a vertical line on paper with square grid or on graph sheet. They label their point of intersection as O (the origin). They then mark and label equal divisions on the horizontal and vertical lines with numbers as shown in Fig 5.32.

We identify the positions of points on the number plane with reference to their distances away from O on the horizontal and vertical lines. We use a pair of numbers to locate a point on the plane. For example, in the plane the point A is (2, 3), where 2 is its distance from O on the horizontal line, and 3 is its distance from O on the vertical line. Similarly, the ordered pairs of numbers that represent the points B, C and D are (4, 1), (6, 6) and (1, 5) respectively.

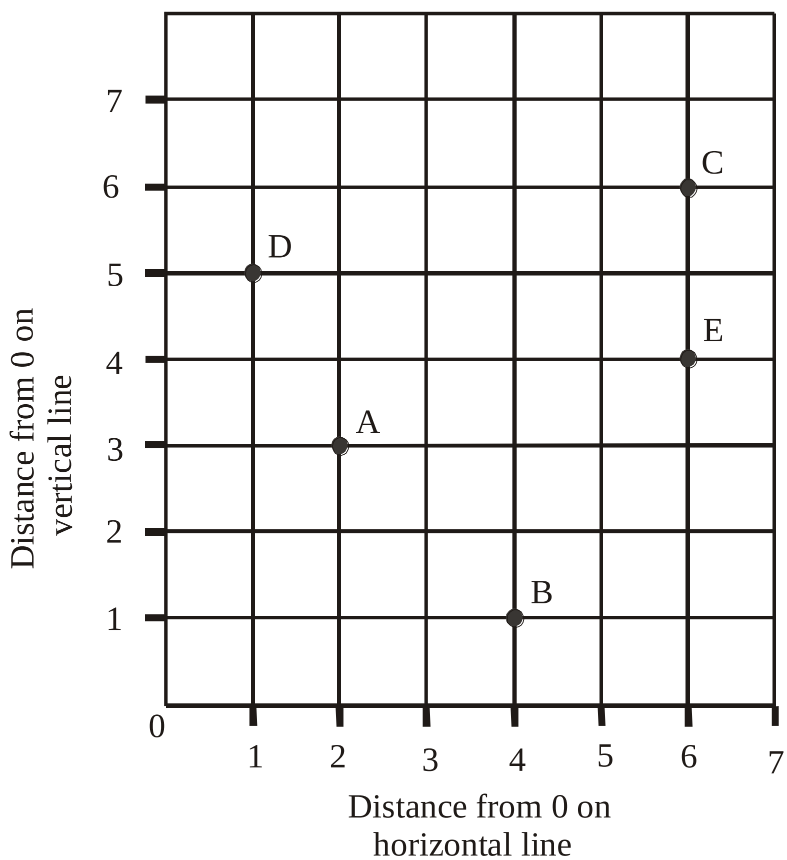


Fig 5.32

Learners practice plotting points on the number plane as shown in Fig. 5.32.

Key Ideas

- Beginners learn about the number plane using the rectangular row and column seating arrangement in the classroom.
- We identify the positions of points on the number plane with reference to their distances away from O, the origin, on the horizontal and vertical lines.
- We use a pair of numbers to locate a point on the plane.

Self-Assessment Questions

1. How should a child correctly describe the location of the object Q in Fig 5.32?
2. A child correctly identifies the point E in Fig5.32 as………………..
3. Draw the number plane and help a Basic 6 child locate the points
   R (1, 3), S (3, 1), and T (5,5) on it.

UNIT 6: TEACHING COLLECTING AND HANDLING DATA

Learners are surrounded by a lot of information they can gather in any environment they live and also in the school environment. In statistics, we refer to most of the information we obtain from our environment as data.

Unit learning outcomes

By the end of this unit, you will be able to assist learners to:

1. Collect and record data by counting and measuring.
2. Organize data into simple frequency tables.
3. Represent data sets using picture graphs/pictographs, block graphs, bar graphs and stem and leaf plots.
4. Read and interpret data from a graph.
5. Find the averages: mode, median, and mean of a data set.

SESSION 1: COLLECTING AND RECORDING DATA

"Collection of data" is the process of obtaining measurements or counts. Data are the observations or information collected either from experiment or survey. At the lower primary level, the teacher should use many objects: bottle tops of different colours, sticks of different sizes and lengths, match boxes, books of different sizes (including exercise books), pens, pencils, milo tins, milk tins, etc.

Session learning outcomes

By the end of the session, you should be able to guide learners to:

1. collect data by counting different objects of the same kind and record them.
2. collect data by measuring lengths/capacity of similar objects and record them.

1.1 Collecting data by counting different objects

In collecting data, learners should collect real objects. Before any collection of data is done, let learners group themselves according to the days they were born and count the learners in each group. Learners can then be led to record this information in their exercise books. For example, in a class of 54 learners, the recording may look like the one below.

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of Learners	6	11	9	7	8	7	6

This information can be written on cardboard and pasted in the classroom. Note that this simple activity is a form of data collection. In Basic 1, let learners collect the items or objects and group them according to their 'kind' or attribute i.e. – colour, size, length, shape etc. and count them to get the number in each group and record them. The recording may be in the form:

Item	Number
1. Pen	12
2. Pencil	14
3. Exercise book	..
etc.	etc

Another example could be asking learners to sort the shapes in Fig.6.1 below according to colour or shape.

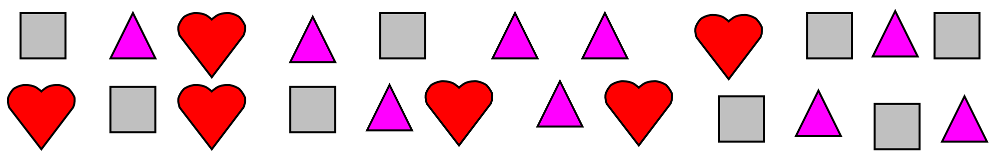


Fig. 6.1

In Basic 2, learners could repeat the Basic 1 activities as a review. Learners could also collect data by sorting, counting, and recording. Example:

(i) find the number of each type of bottle tops in a collection of bottle tops.

(ii) find the number of learners who are taller/shorter than or same as a given height.

(iii) find the number of items that satisfy given criteria and record the results, e.g., pens/pencils/sticks that are longer/shorter than or same as a given length, or containers that can hold same or more/less amount of water as a given container.

In Basic 3, let learners perform an activity and record the data involved. For example, tossing a coin a given number of times and recording the number of times each side shows up. [Example, a coin tossed 10 times, and side with coat of arms appears 6 times and the other side appears 4 times].

1.2 Collecting data by measuring lengths/capacity of similar objects

In Basics 4 to Basic 6, let learners repeat the activities done by the Basic 3 learners. Let them continue by collecting data and recording by counting [activities (i) – (iii)] in the examples below:

(i) Throwing a die a given number of times and recording the number of times each face shows up.

(ii) Writing the names of ten learners and recording how many times some given vowels appear in their names.

(iii) Recording the marks obtained in a class exercise marked out of ten and finding the number of learners who scored the given marks.

Lead leaners to now collect data by measuring using the examples below:

(iv) Let learners find their heights with a measuring tape and record the number of learners within intervals of heights. For example, learners with heights between 80*cm* – 100*cm*; 100*cm* – 120*cm*; 120*cm* – 140*cm* etc.

(v) Measuring masses of learners and finding the number within given measures e.g. number of learners weighing 20*kg* – 25*kg*, 26*kg* – 31*kg*, etc.

Key Ideas

- Data are the observations or information collected either from an experiment or a survey.
- Collection of data is done through counting and measuring.
-

Self –Assessment Questions

1. Design an instrument which you may use to collect the following data about 50 learners in a school of your choice:
   a. Birth days
   b. Birth months
   c. Heights
   d. Masses
   e. Favorite foods

SESSION 2: ORGANIZING DATA INTO FREQUENCY TABLES

Two methods of presenting collected quantitative data are in use. One method involves a summarized presentation of the data themselves, usually in tabular form. The other consists of presenting the data in pictorial form (graphs). The latter form will be discussed in the Sessions 3 and 4.

Session learning outcomes

By the end of this session you should be able guide learners to organize data into:

1. ungrouped frequency tables
2. grouped frequency tables

2.1 Organising data into ungrouped frequency tables

In this session, we consider organisation of data. The collected data is called *raw data*. For example, marks obtained by 15 learners in a class exercise marked out of ten may look like:

2 6 1 8 5 9 3 5 8 3 9 7 4 2 6.

These scores are not organized. It is an example of a **raw data**. To record these scores statistically, i.e. organizing the data, it is often advisable to rewrite the data in order of magnitude in a *frequency count* form. We can, therefore, organize the above data as follows:

Marks	1	2	3	4	5	6	7	8	9
No. of learners	1	2	2	1	2	2	1	2	2

Note that the marks range from 1 to 9. Hence the numbers 1 to 9 are recorded as marks. The frequencies of the marks indicate the number of learners who obtained the scores. Thus the second row in the table is recorded as “No. of learners”. This can also be recorded as “frequency”. Instead of the above *horizontal* or row table, some people draw *vertical* or column table. Note that the horizontal table saves a lot of space for other work to be done.

2.2 Organising data into grouped frequency tables

With the upper primary let learners be aware that, at times, the collection of information may lead to very large masses of data that it may be tiresome to consider the individual datum as the one above. In such situations, we put the data into groups called *grouped data*.

Example: Heights (in centimeters) of 30 learners in a class are as follows:

145	150	147	152	155	148	149	151	155	162
140	139	136	158	154	152	147	149	142	137
163	145	153	158	153	154	159	160	153	152

Using intervals 135 – 140, 141 – 146, 147 – 152 etc. draw a table of frequency of the data.

To answer this question, we draw a table, using the given intervals, and find the number of learners who have their heights falling within each interval.

Heights (*cm*)	135 – 140	141 – 146	147 – 152	153 – 158	159 – 164
Tally	////	///	~~////~~ ~~////~~	~~////~~ ////	////
No. of Learners	4	3	10	9	4

In Basics 4 to 6, where learners are to measure their heights or masses, the data derived from these activities could be put in grouped form.

Key Ideas

- Two methods of presenting collected quantitative data usually in tabular form and in pictorial form (graphs).
- The tabular form of data representation is called frequency distribution table
- Graphical forms of data representations are: pictographs. Block graphs, bar graphs, histogram, pie chart stem, and leaf plot.

Self – Assessment Questions

Organise data obtained into unit 6 session 2 above into the following representations:

1. Ungrouped frequency tables
2. Grouped frequency tables

SESSION 3: REPRESENTING DATA

In Sessions 1 and 2, we learnt how to guide learners to collect and organize data into frequency tables. In this session we shall learn how to present data in diagrammatic form called *graph.*

Session learning outcomes

By the end of the session, you should be able to guide learners to:

1. represent data as simple block graph
2. represent data using pictogram and pictograph
3. represent data using bar graph
4. represent data using stem-and-leaf plot

3.1 Representing data as simple block graphs

In Session 2, we stated that one method of representing quantitative data is by means of diagram or simply graphs. We now consider some simple ways of representing data diagrammatically. Representation of data on a graph can be started in Basic 1.

For example, 20 learners, each was asked to throw a die and record the number that shows up. The results are as follows:

2	5	1	6	3	4	6	3	2	5
3	4	2	6	5	2	3	1	6	3

In Basic 2, learners could draw squares or rectangles to represent the above results.
Before drawing the squares or rectangles, learners need to be guided to organize the data as shown below:

Score	1	2	3	4	5	6
No. of Learners	2	4	5	2	3	4

Then, learners are guided to use the squares or rectangles to draw the block graph as shown in Fig.6.2 below.

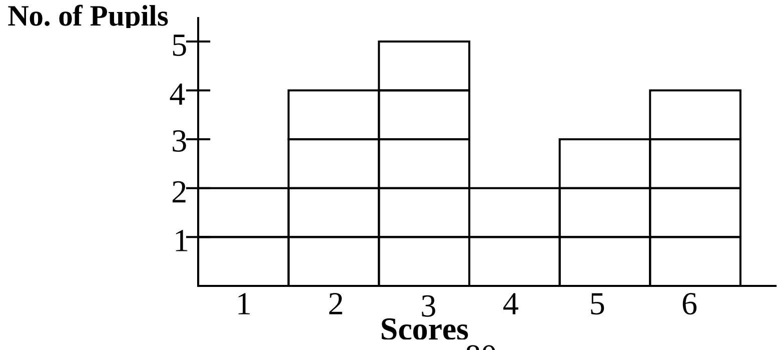


Fig. 6.2

A square or a rectangle represents a pupil. Thus, since 2 learners scored 1 in the throw, two rectangles are drawn on 1. Similarly, 4 learners scored 6 in the throw, hence four rectangles are drawn on 6. To make the graph more attractive since learners are attracted by colours, learners can colour the rectangles as shown in Fig. 6.3.

Fig. 6.3

The above graph is called *block graph* because the squares or rectangles seem to form blocks with no spaces between them. The same graph can be taught in Primary 3 and 4. Note that the vertical axis is labeled *No. of Learners* – i.e. the frequency (this is sometimes labeled *frequency*) and the horizontal axis, *scores*, the numbers the learners scored during the throws. These are the labels of the graph. Let learners indicate how they come to school – that is by *walking*, by *taxi*, by *car* and by *bicycle*. Let them draw a block graph representing the information they have given. Again, let learners indicate which of the following types of food they like best: *Fufu*, *Fried rice*, *Red-red* and *Ampesi*. Let them draw the block graph for the data obtained. They could then colour their graphs.

3.2 Representing data using pictogram and pictograph

A pictogram (picture graph) represents data in the form of pictures, objects or parts of objects. This representation of pictures or objects may be done by cutting and pasting pictures of real things; drawing pictures to represent real objects or use some model to stand for the objects in picture graphs. For example, in a class of 30 learners made up of 18 girls and 12 boys sitting in 5 rows, the population of the class can be represented pictorially as in Fig.6.4.

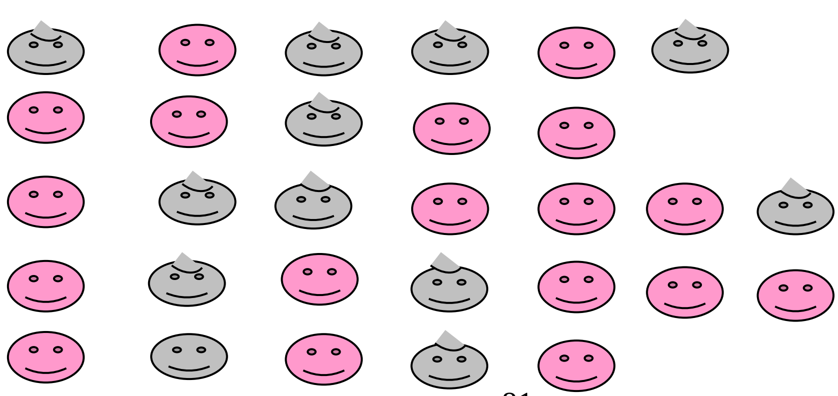
Example 1

ROW 1

ROW 2

ROW 3

ROW 4



ROW 5

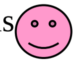
Where the models  and  represent a girl and a boy respectively.

Fig. 6.4

In the class above, how many girls are in each row?

Let learners cut some pictures of real things from magazines and display or present as data. Let learners also draw objects and colour to represents data and display. For instance, the number of cars that come to the school from Monday to Friday, the games they like best etc.

Example 2

Pictograph is similar to the picture graph we considered in Sub-session 3.2. This type of representation is common in government pamphlets and in advertisements. Such presentations register a meaningful impression in our mind almost before we think about them. They are useful in capturing the attention of persons who ordinarily would not look at more formal chart of graph, hence its use at this level.

Example is presented in Fig.6.5, where the data shows the number of cars that were bought in a certain town in Ghana from 1995 to 2000. Use the data to orally answer the questions that follow.

The data shows the number of cars bought in a certain town in Ghana each year from 1995 to 2000.

Each of the  represents 10 cars

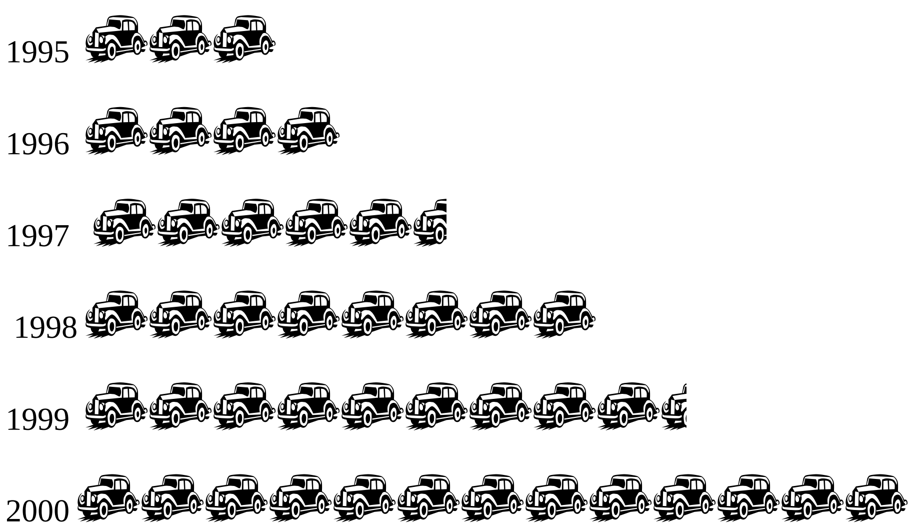


Fig. 6.5

3.3 Representing data using bar graph

Review representation of data using block graph with Primary 5 and 6 learners. Let learners draw or colour squares to represent numbers of objects, people or quantities measured. For example, *number of learners* and the *days they were born*, or *number of learners* and *their ages* (in years). Note that with the block graph, there are no spaces between the '*rectangles*'. We can also represent data in a bar graph. For example, the data for the 20 learners who threw a die and recorded the scores on the die presented in Sub-session 3.1 can be represented by a bar graph. We reproduce the data. The results were as follows:

2	5	1	6	3	4	6	3	2	5
3	4	2	6	5	2	3	1	6	3

Let us now represent this information using bar graph. Before then, we have to organize the data. Organizing it gives:

Score	1	2	3	4	5	6
No. of Learners	2	4	5	2	3	4

The bar graph is drawn and shown in Fig. 6.6 below.

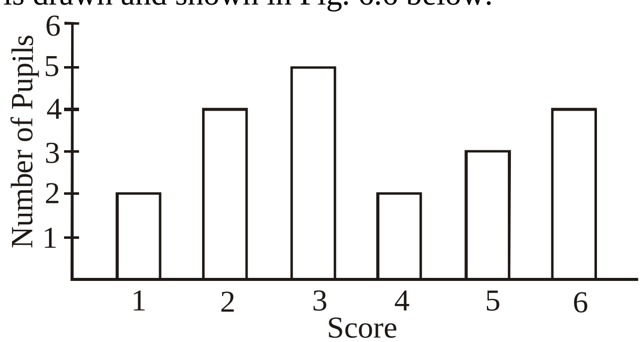


Fig. 6.5

Note that unlike the block graph, the bar graph has spaces between the bars. The bars are equally spaced. Like the block graph, the bars may be coloured just to make it more attractive to learners. Let learners draw bar graphs to represent numbers of objects, people and quantities measured.
For example, *number of learners* and the *days* they were born, or the *number of learners* and their *shoe sizes*, or *the number of* learners and their *heights* or *ages*, etc.

Another example could be the graph of learners showing the type of food they like best.

The graph in Fig.6.6 below shows the type of food some learners like best. Use it to orally answer the questions below it.

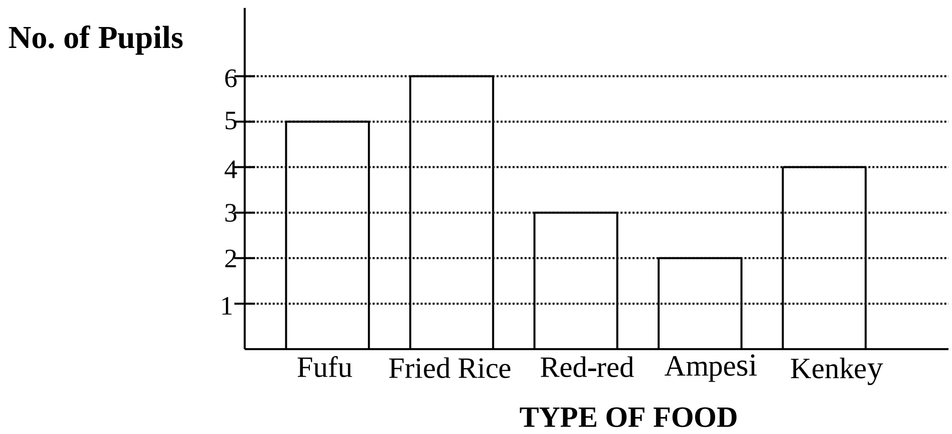


Fig. 6.6

Now answer the following questions.

1. How many learners like Fried rice?
2. How many learners say they like Kenkey?
3. How many learners like Ampesi and red-red?
4. How many learners were involved in the survey?

3. 4 Representing data using stem-and-leaf plot

A *stem-and-leaf plot* is a quick numerical method of providing a visual summary of data. This method suggests the *stems* of plants and their leaves. Consider the following test scores of 18 learners marked out of 36.

12	23	14	35	29	25
34	31	16	27	25	16
26	25	19	20	21	18

To draw a stem-and-leaf plot of this data, we first observe that the scores range from 10s to 30s. The tens digits of 1, 2 and 3 are chosen as the stems, and the unit digits of the numbers represent the leaves. The first step in forming a stem-and-leaf plot is to list the stem values in increasing order in a column

[Fig 6.7 (*a*)]. Next, each leaf value is written in the row corresponding to that number's stem. Here, the leaf values are recorded in the order in which they appear in the data. However, the leaf values are finally listed in increasing order [Fig 6.7 (*b*)].

Stem	Leaf
1	
2	
3	

Stem	Leaf
1	2 4 6 6 8 9
2	0 1 3 5 5 5 6 7 9
3	1 4 5

(*a*) **(*b*)**

Fig. 6.7

Another example is the following test marks of 26 learners:

82	66	70	77	94	67	73	78	82	74	90	45	62
85	57	72	94	83	85	70	95	71	89	87	75	74

To represent these scores using stem-and-leaf plot, we note that the marks range from 40s to 90s. The tens digits are 4, 5, 6, 7, 8 and 9 and these are the stems, and the unit digits of the numbers represent the leaves. The stem-and-leaf plot of the scores is drawn in Fig.6.8 below.

Stem	Leaf
4	5
5	7
6	2 6 7
7	0 0 1 2 3 4 4 5 7 8
8	2 2 3 5 7 9
9	0 4 4 5

Fig. 6.8

When the numbers to be represented, using the stem-and-leaf plot are 3-digits, we normally use the hundreds and tens as the stem and the units as the leaves. For example, to represent the following 18 numbers

112	123	114	135	129	125	134	131	116
127	125	116	127	125	120	121	122	118

on a stem-and-leaf plot, we notice that the first two digits range from 11 to 13. The numbers 11, 12 and 13 form the stem. The stem-and-leaf plot is presented in Fig.6.9 below.

Stem	Leaf
11	2 4 6 6 8
12	0 1 2 3 5 5 5 7 7 9
13	1 4 5

Fig. 6.9

Key Ideas

- A block graph is a pictorial representation of a data set using rectangular shapes or square shapes for blocks to represent the total occurrence (frequency) a number or object in the data set.
- A pictogram (picture graph) represents data in the form of pictures, objects or parts of objects
- A *stem-and-leaf plot* is a quick numerical method of providing a visual summary of data

Organise data obtained into unit ^ session 2 above into the following representations:

1. a. into pictograph
2. b. into block graphs
3. e. into bar graph
4. d. into stem and leaf plots (Use 1 decimal place masses)
5. c. into histogram

SESSION 4: READING AND INTERPRETING GRAPHS

The oral exercises under sub-sessions 3.1 and 3.3 are meant to prepare you for this session. At times, data may be presented in the form of a diagram – i.e. any of the graphs we have discussed so far. In this session, we shall learn to read and interpret data from a given graph.

Session learning outcomes

By the end of the session, you should be able to guide learners to:

1. read data and interpret data from graphs

5.1 Reading data and interpreting graphs

When data is presented in a graph form, we must study the graph carefully, taking into consideration, the label as well as any key given to the graph. For example, let us read data from the bar graph presented in sub-session 3.1 on the types of food learners like best. The graph is thus reproduced in Fig.6.10.

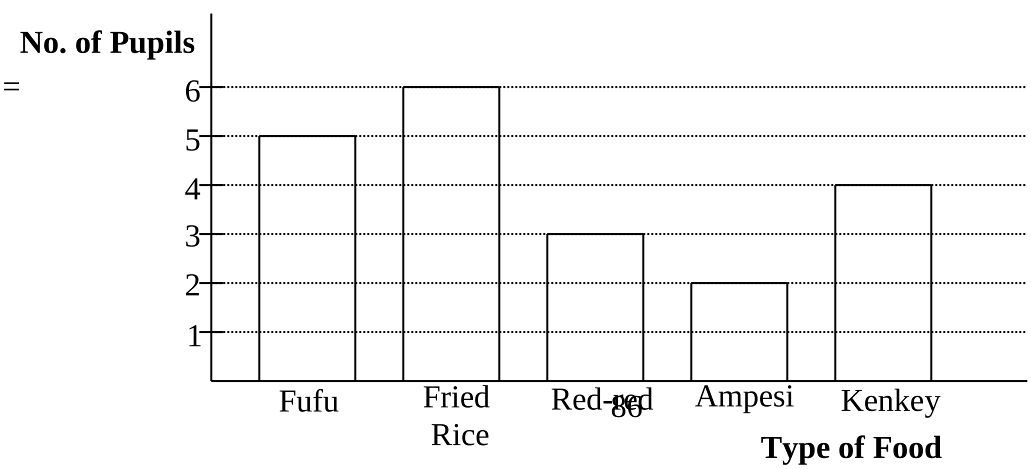


Fig. 6.10

Studying the graph carefully will reveal that the types of food the survey covered were Fufu, Fried rice, Red-red, Ampesi and Kenkey. (5 types of food). The vertical axis is the frequency axis (or number of learners). There were 5 learners who said they like fufu best. For fried rice, 6 learners opted for it and 3 learners opted for Red-red. While 2 learners opted for Ampesi, 4 learners said they like kenkey. Hence, in all, there were 20 learners that were involved in the survey. These are all the pieces of information we can read from the above graph.

To teach this then, the teacher has to draw some of these graphs on the chalkboard or on a cardboard and display on the chalkboard for class discussion through questioning by the teacher. We can easily write down the numbers in a stem-and-leaf plot. For example, using the stem-and-leaf plot in Fig.6.11 below, write down the numbers that were used to plot it.

Stem	Leaf
21	0 1 5 8
22	0 1 2 2 4 7 8 9
23	0 1 3 5 7 7
24	7 8

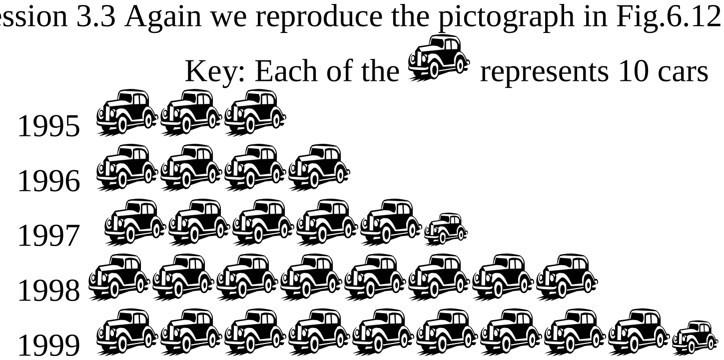
Fig. 6.11

A careful look at the diagram will reveals that, the numbers range from 210s to 240s. Indeed, the numbers are:

211	215	210	218	222
224	227	228	220	221
229	222	230	231	237
237	235	233	247	248

We see that it is easy to read all the 25 numbers at a glance on the stem-and-leaf plot.

Another interesting and quite easy graph to read is the picture graph or pictograph. Let us revisit the picture graph of the cars bought from 1995 to 2000 in a certain town in Ghana, presented in sub-session 3.3 Again we reproduce the pictograph in Fig.6.12.



2000 

Fig. 6.12

In this picture graph, a key is given as a  represents 10 cars.

Counting the number of cars bought for each year in the town gives the following data.

Year	1995	1996	1997	1998	1999	2000
No. of Cars	30	40	55	80	95	130

Note that the number of cars bought each year is multiplied by 10. This is because of the key. Another interesting feature of this picture graph is the one-half car for the last picture for 1997 and 1999. Half of ten is five, and this is why there were 55 and 95 cars bought in 1997 and 1999 respectively.

Key Ideas

- When data is presented in a graph form, we must study the graph carefully, taking into consideration, the label as well as any key given to the graph

Self-Assessment Questions

SESSION 5: AVERAGES: MODE, MEDIAN, MEAN

So far, we have considered data collection and organization of the data. Once data have been collected and organized, we should be able to understand and communicate the results to others. For example, if a learner records the amount of time spent watching television each day for a 10-day period, these 10 numbers can be replaced by a '*typical*' number or a '*central*' number to describe the amount of time in general that learners' watch television. In this session, we shall consider some three of such numbers called *mode*, *median* and *mean* (also called *averages*). The mode and median are introduced in Basic 5 while mean is introduced in Basic 6.

Session learning outcomes

By the end of the session, you should be able guide learners to:

1. find the mode of a set of numbers or objects
2. find the median of a set of numbers
3. find the mean (average) of a set of data.

5.1 Finding the mode of a set of numbers

Finding mode or median is introduced in Primary Class 5. In guiding learners to find the mode of a set of numbers, let them give you, their ages. Write the ages on the chalkboard as they give you.

You will realize that most of the learners are about 11 years, though few may be 10 years or 12 or even 13. The age of the majority of them (*the most typical age*) is what we call the **mode.** If in the class, majority of the learners are 11 years old, then the *modal age in the class is 11 years.* For example, if a class of 20 learners, their ages are given as follows:

11	12	13	11	13	12	11	11	12	11
12	13	11	13	11	12	12	11	11	13

we realize the most typical or common age is 11. So, the mode of the set of numbers above is 11.

The stem-and-leaf plot may also be used to find the mode of a set of numbers. For example, in Fig. 6.13 below, the number that occurs most is 23.

Stem	Leaf
1	1 2 6
2	2 3 3 3 8
3	2 4 5

Fig. 6.13

5.2 Finding the median of a set of numbers

As the name indicates, median is the central number of a set of data. However, the central number can be arrived at only when the numbers have been arranged in order of magnitude. For example, to find the median of the numbers 2, 4, 3, 5, 2, 7, 8, 7, and 2, we first rearrange them in order of magnitude as 2, 2, 2, 3, 4, 5, 7, 7, 8.

There are nine numbers in all, four to the left of 4 and four to the right of 4. Hence the **median** is 4. Here, the number of the items is odd (i.e. 9) and selecting the middle number leaves equal number of items on each side of the middle one. Another example is to find the median of the set of numbers: 12, 22, 16, 11, 23, 28, 23, 32, 23, 35, 34, 37, 38. Again, we rearrange the numbers in order of magnitude as:

11, 12, 16, 22, 23, 23, | 23, | 28, 32, 34, 35, 37, 38.

Again, the middle number is 23.

Note: The arrangement of the numbers in order of magnitude can be in ascending order (as in the examples above) or descending order.

In the above examples, the numbers of items involved are all odd. When the number of items involved is even, then we find the median as the average of the two middle numbers when the numbers are arranged in order of magnitude.

For the set of the numbers 12, 22, 16, 11, 23, 28, 23, 32, 32, 35, 34, 37, 38, 39, the number of items involved is 14 (which is an even number). To find the median of these numbers we first rearrange them in order of magnitude.

39, 38, 37, 35, 34, 32, 32, 28, 23, 23, 22 16, 12, 11

The median should be at the middle place where the vertical thick line is drawn. But there is no number there, hence we find the average of the two middle numbers i.e. 28 and 32. Therefore the median = $\frac{28+32}{2} = \frac{60}{2} = 30$.

Let learners find the median age of the class.

5.3 Finding mean of a set of data

In session 6.2, we saw that the average of two numbers is given by the sum and dividing the sum by two. Extend this idea by asking learners to find the average age of 2, 3, 4 and 5 learners in the class. After going through these activities, lead learners to realize that to find mean or average of a set of numbers, we first find the sum and divide the sum by the number of items involved. For example, the mean of the set of numbers:

2, 2, 3, 5, 6, 6, 7, 9 is given by

$$\text{mean} = \frac{2+2+3+5+6+6+7+9}{8} = \frac{40}{8} = 5.$$

Let learners find their mean age and mean height.

Key Ideas

- The mode of a data set of numbers or objects is the most popular number of object of the data set.
- The median is the central number or the average of the central numbers of a set of data when the numbers in the data set are arranged in order of magnitude.
- The mean of a data set of numbers is the ratio of the sum of the numbers in the data set to the number of numbers in the data set.

Self-Assessment Questions

1. **Describe how you would assist a B6 learner to find the (a) Mode, (b) Median and (c) Mean of the data set of scores below:**

   1 4 5 3 2

   3 2 4 3 5

   4 1 5 4 4

   3 4 3 5 4
2. **How would you assist a B6 learner to find the (a) mean age, (b) median, and (c) modal age of ten of his/her friends.**